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ELECTRICAL/ELECTRONICS ENGINEERING

ELECTROMAGNETIC WAVES (PEP 318)

4) from $\frac{d^2 E_y}{dx^2} = (j\omega M \epsilon - \omega^2 M \epsilon) E_y$

where $\rightarrow \frac{d^2 E_y}{dx^2} = r^2 E_y$

$r = \alpha + j\beta$

$E_y = \epsilon_0 e^{-\alpha x} = \epsilon_0 e^{-\omega x} e^{-j\beta x}$

$\frac{d^2 E_y}{dx^2} = j\omega M \epsilon E_y = r^2 E_y$

$r = \underline{r^2} = \alpha + j\beta$

$\sqrt{j\omega M \epsilon}$

from $\sqrt{j} = \frac{1 + j}{\sqrt{2}}$

we have; $r = \sqrt{\frac{\omega M \epsilon}{2}} + j \sqrt{\frac{\omega M \epsilon}{2}}$

$\alpha = \sqrt{\frac{\omega M \epsilon}{2}} \quad \beta = \sqrt{\frac{\omega M \epsilon}{2}}$

$\therefore E_y = \epsilon_0 e^{-\frac{\omega M \epsilon}{2} x} e^{-j \frac{\omega M \epsilon}{2} x}$

we have; $E_y = \epsilon_0 e^{-\alpha x} e^{-j\beta x}$

This indicates that the amplitude of the wave decreases exponentially as it penetrates a conductive medium by a factor $e^{-\alpha x}$

b) Skin depth is the depth of penetration of a wave inside a conductor.

$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$ and $\omega = 2\pi f$

$= \sqrt{\frac{2}{2\pi f \mu \sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}}$

$\therefore \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$ or $\sqrt{\frac{1}{\pi f \mu \sigma}}$

c) $f = 10 \text{ MHz} \approx 1 \times 10^7 \text{ Hz}$

$$r = 5.8 \times 10^{-7} \text{ dm}, N_r = 1, N_0 = 1.257 \times 10^{-6}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu_r}} = \frac{1}{\sqrt{\pi f \mu_r}}$$

$$M = \mu_r \times M_0 = 1 \times 1.257 \times 10^{-6} = 1.257 \times 10^{-6}$$

$$\delta = \frac{1}{\sqrt{\pi \times 1 \times 10^7 \times 1.257 \times 10^{-6} \times 5.6 \times 10^{-7}}}$$

$$\delta = 2.09 \times 10^{-6} \text{ m}$$

7) $b = 10 \text{ mm} = 0.01 \text{ m}$, $a = 3 \text{ mm} = 0.003 \text{ m}$
 $\epsilon_{r0} = 8.85 \times 10^{-12}$, $M_0 = 1.257 \times 10^{-6}$

a) Capacitance per meter

$$C = \frac{2\pi \epsilon_0 b}{\log \frac{b}{a}}$$

$$C = \frac{2\pi \times 8.85 \times 10^{-12}}{\log \frac{0.01}{0.003}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\log 28.03}$$

$$\therefore C = 3.84 \times 10^{-4} \text{ f/m}$$

b) Inductance per meter

$$L = \frac{\mu_0}{2\pi} \log \frac{b}{a}$$

$$= \frac{1.257 \times 10^{-6}}{2\pi} \times \log \frac{0.01}{0.003} = \frac{1.257 \times 10^{-6}}{2\pi} \times \log 28.03$$

$$\therefore L = 2.90 \times 10^{-7} \text{ H/m}$$

c) Characteristic Impedance

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{2.90 \times 10^{-7}}{3.84 \times 10^{-4}}} = 86.90 \Omega$$

d) Phase velocity

$$V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.9 \times 10^{-7} \times 3.84 \times 10^{-4}}} = 29.10 \times 10^7 \text{ V/m}$$