Modilim Tochukwu Adrian 18th - April 2020
17/Eng04/041 Sign: TAModilim
Electrical/Electronics Engineering
Electromagnetic wave theory
EEE 316 ASSIGNMENT 1

7a Capacitance per meter, C

Recall = $C = \frac{2\pi\varepsilon_0}{\log_e b/a}$

$= \frac{2 \times 3.142 \times \varepsilon_0}{\log_e b/a}$

$\therefore b = 10mm \Rightarrow 0.01m \quad a = 3m \Rightarrow 0.003m$

$\varepsilon_0 = 8.85 \times 10^{-12} f/m, \; \mu_0 = 1.287 \times 10^{-8} H/m$

$C = \frac{2 \times 3.142 \times 8.85 \times 10^{-12}}{\log_e 0.01/0.03}$

$\therefore C = 3.84 \times 10^{-11} f/m$

b) Inductance per meter, C

$L = \frac{\mu_0}{2\pi} \log_e b/a$

$L = \frac{1.257 \times 10^{-6}}{2 \times 3.142} \times \log_e 0.01/0.03$

$L = 2.90 \times 10^{-7} H/m$

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c) Characteristic impedance Z

$Z_0 = \sqrt{\frac{L}{C}}$

$Z_0 = \sqrt{\frac{2.9\times10^{-6}}{3.84\times10^{-11}}}$

$Z_0 = 86.90\,\Omega$

d) phase Velocity V_p

$V_p = \frac{1}{\sqrt{LC}} \Rightarrow \frac{1}{\sqrt{2.9\times10^{-7} \times 3.84\times10^{-11}}}$

$V_p = 2996\,4500.0$

$V_p = 3\times10^{8}\,m/s$ (approx speed of light)

4) from $\frac{d^2E_y}{dx^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y$

We have $\rightarrow \frac{d^2E_y}{dx^2} = \gamma^2 E_y$

$\gamma = \alpha + j\beta$

$E_y = E_0 e^{-\gamma x} = E_0 e^{-\alpha x} e^{-j\beta x}$

$\frac{d^2E_y}{dx^2} = j\omega\mu\sigma E_y = \gamma^2 E_y$

$\gamma^2 = j\omega\mu\sigma$

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$\gamma = \sqrt{j\omega\mu_0} = \alpha + j\beta$

from $\sqrt{j} =$

$\mathcal{E}_y = \mathcal{E}_0 e^{-\gamma x} = \mathcal{E}_0 e^{-\alpha x} e^{-j\beta x}$

Suppressing the $e^{j\omega t}$ terms) - for a wave travelling in a conducting medium where conductivity $\sigma >> \omega\varepsilon_0$

$$\frac{d^2\mathcal{E}_y}{dx^2} = j\omega\mu_0\sigma\mathcal{E}_0 = \gamma^2\mathcal{E}_y \quad -(1)$$

$\gamma^2 \; j\omega\mu\sigma$

$\gamma = \sqrt{j\omega\mu\sigma}$

for the real and imaginary part of γ that we have

$\gamma = \sqrt{}$

$$\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\mathcal{E}_y = \mathcal{E}_0 e^{\sqrt{\frac{\omega\mu\sigma}{2}}x} \; e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x}$$

$$\mathcal{E}_y = \mathcal{E}_0 e^{-x/\delta} e^{-jx/\delta}$$

shows amplitude of wave decreases exponentially as it penetrates a conducting medium by a factor of $e^{-x/\delta}$

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b) Skin depth is defined as depth of penetration of wave inside a conductor

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

$\therefore \omega = 2\pi f$

$$\delta = \sqrt{\frac{2}{2\pi f \mu \sigma}} = \frac{1}{\pi f \mu \sigma}$$

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$$

c) $f = 10MHz \Rightarrow 1 \times 10^7 Hz$

$\sigma = 5.8 \times 10^9$ S/m $\mu = 1$ $\mu_0 = 1.257 \times 10^{-6}$

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} =$$

$\mu = \mu_r \times \mu_0 = 1.257 \times 10^{-6}$

$$\delta = \frac{1}{\sqrt{3.142 \times 1 \times 10^7 \times 1.257 \times 10^{-6} \times 5.8 \times 10^7}}$$

$\delta = 2.09 \times 10^{-5}$ m