17Eng04l014
EEE 316

QUESTIONS 4 & 7

4. If the medium through which the em wave propagates is a good
(a) Analyse the equation in Q3 to show that the wave amplitude de
exponentially as its penetrates the medium.
(b) Define the depth of penetration known as the skin depth and
its value in terms of the parameters of the medium and the frequ
of the signal
(c) Calculate the depth of penetration of the wave in a sheet of
at a frequency of 10 MHz at which the wave amplitude decreases
1% of its value upon entering the sheet.
Take $\sigma = 5.8 \times 10^7$ s/m, $\mu_r = 1$ for copper.

Solution.

(a)
$$\frac{\delta^2 E_y}{\delta x^2} = [j\omega\mu\sigma - \omega^2\mu\varepsilon] E_y$$

Analyse the Equation.

$$\frac{\delta^2 E_y}{\delta x^2} = [j\omega\mu\sigma - \omega^2\mu\varepsilon] E_y$$ Eqn i → An Equation in em wave prop in material medium.

$$= \gamma^2 E_y \quad \text{--- Eqn (ii)}$$

where ; γ = Propagation constant
Note that :
Propagation in a good conducting medium Neglect $\omega^2\mu\varepsilon$
$\therefore \sigma >> \omega^2\mu\varepsilon$

eq(ii) becomes
$$\frac{\delta^2 E_y}{\delta x^2} = [j\omega\mu\sigma] E_y$$

$$\therefore \gamma^2 = j\omega\mu\sigma$$

$$\gamma = \sqrt{j\omega\mu\sigma} \quad \text{--- eqn iii}$$

Eqn (iii) becomes:

$$\gamma = \frac{1}{\sqrt{2}} \cdot \sqrt{\omega\mu\sigma} + \frac{1}{\sqrt{2}} \cdot j\sqrt{\omega\mu\sigma} \quad \text{--- Eqn (v)}$$

Recall that; $\gamma = \alpha + j\beta$

Referring to Equation (v).

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{and} \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \quad \text{--- Eqn (vi).}$$

Put Eqn (vi) into Eqn (iv)

$$E_y = E_0 e^{-\alpha x} \cdot e^{-j\beta x}$$

$$E_y \Rightarrow E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} \cdot e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x} \quad \text{--- Eqn (vii)}$$

where δ; depth of penetration / skin depth $= \sqrt{\frac{2}{\omega\mu\sigma}}$

where $\omega = 2\pi f$

$$\delta = \sqrt{\frac{2}{2\pi f\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

Equation vii becomes

$$E_y = E_0 e^{-x/\delta} e^{-jx/\delta}$$

amplitude wave decreases exponentially as it penetrates into a conducting medium by a factor $e^{-x/\delta}$

$\therefore$ when $x = \delta$; $e^{-x/\delta} = e^{-1}$

; waves decreases by $e^{-1} = 0.37$ of its value to each [illegible] the conducting medium.

Expressing γ as a complex parameter in terms of real and imaginary part;

$$\gamma = \alpha + j\beta \quad --- \quad Eq\ (iv)$$

α = Real, $j\beta$ = Imaginary part.
α = the attenuation constant in nepor m^{-1}
β = the phase constant in radian m^{-1}

A solution of Eqn (ii) of a wave travelling in a dielection is

$$E_y = E_0 e^{-\gamma x} = E_0 e^{-\alpha x} \cdot e^{-j\beta x} \quad -- \quad Eqn\ (iv)$$

From Equation (iii)

$$\gamma^2 = j\omega\mu\sigma$$

$$\gamma = \sqrt{j\omega\mu\sigma}$$

$\therefore$ Recall that;

$$\sqrt{j} = \frac{1+j}{\sqrt{2}}$$

Proving this;

R.H.S = $\sqrt{j} \Rightarrow (\sqrt{j})^2 = j$

$$LHS = \frac{1+j}{\sqrt{2}} = \left[\frac{1+j}{\sqrt{2}}\right]^2 = \frac{1+2j+j^2}{2} \quad \text{where } [j^2] = -1$$

$$\therefore \frac{1+2j-1}{2} = \frac{2j}{2} = j //$$

It has been proved that $\sqrt{j} = \frac{1+j}{\sqrt{2}}$

Using $\sqrt{j} = \frac{1+j}{\sqrt{2}}$ Applying to Eqn (iii).

$$\sqrt{j} = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}.$$

Calculate the depth of Penetration.

$$\delta = 1 / \sqrt{\pi f \mu \sigma}$$

$$\delta = \frac{1}{\sqrt{\pi \times 10 \times 10^{6} \times 5.8 \times 10^{7} \times 4\pi \times 10^{-7}}}$$

$$\delta = 2.09 \times 10^{-5} m$$

for the depth increase of $1\% = 10^{-2}$

$$e^{-x/\delta} = 10^{-2}$$
$$\ln = [e^{-x/\delta}] = \ln(10^{-2}]$$
$$-x/\delta = -4.61$$

$$-x = -4.6\delta$$
$$x = 4.6\delta$$

where $\delta = 2.09 \times 10^{-5}$
$$x = 4.6 \times 2.09 \times 10^{-5}$$
$$x = 9.6 \times 10^{-5} m.$$

The 1 % depth decrease $= 9.6 \times 10^{-5} m$

4(b)

Depth of Penetration/Skin depth;

This is the distance at which the wave reduces or decays to 37% of E_o when moving from medium 1 to medium 2. Skin depth is dependent on the frequency the higher the frequency the lower the skin depth.

$$\delta = \frac{1}{\sqrt{\pi f \mu_o \sigma}}$$

Deriving it values at different frequency

For copper $\mu_r = 1$, $\mu_o = 4\pi \times 10^{-1}$, $\sigma = 58 M\mu m^{-1}$

$$\delta = \frac{6.6 \times 10^{-2}}{\sqrt{f}}$$

For specific frequencies

at 50 Hz , $\delta = 8.5 \times 10^{-3}$ m

at 3 mHz ; $\delta = 3.8105 \times 10^{-5}$ m

at 30 GHz , $\delta = 3.8 \times 10^{-7}$ m.

at 200 MHz , $\delta = 4.66 \times 10^{-7}$

at 100 GHz ; $\delta = 6.6 \times 10^{-22}$

$c = 4.62 \times 10^{-11}\ Fm^{-1}$

(b) Inductance per meter

$$L = \frac{\mu_0}{2\pi} \log_e b/a$$

$$L = \frac{4\pi \times 10^{-7}}{2\pi} * \log_e \left[\frac{0.01}{0.003}\right]$$

$L = 2.4 \times 10^{-7}$ Henry/m.

(c) Characteristic impedance, Z_0

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{2.4 \times 10^{-7}}{4.6 \times 10^{-12}}}$$

$Z_0 = 72.23\,\Omega$

(d) Phase velocity

$$v = 1/\sqrt{LC}$$

$$v = 1/\sqrt{2.4 \times 10^{-7} \times 46 \times 10^{-12}}$$

$v = 30.10 \times 10^{7}\ ms^{-1}$ //.

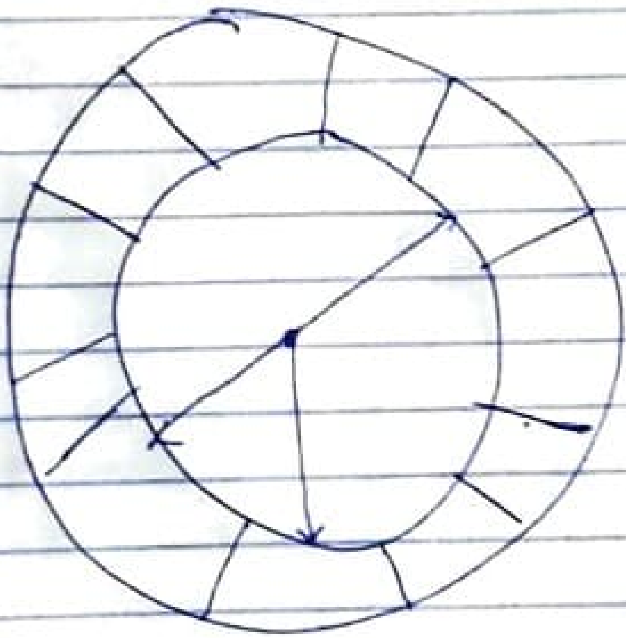
outside diameter, a = 3mm Calculate the

(a) Capacitance per meter, C
(b) Inductance per meter, L
(c) Characteristic Impedance, Z_0
(d) Phase velocity, V_p of an Em wave propagate through it

Hint : $C = \frac{2\pi\varepsilon_0}{\log_e b/a}$ $L = \frac{\mu_0}{2\pi}\log_e b/a$

Solution

+E



(a) Capacitance per meter, C

diameter = $a = 3mm = 0.003m$

$b = 10mm = 0.01m$

Using the formula $C = \frac{2\pi\varepsilon_0}{\log_e b/a}$

$C = \frac{2 \times \pi \times 8.85 \times 10^{-12}}{}$