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ELECT /ELECT

from $\frac{d^2 E_y}{dx^2} = (j\omega\mu\sigma - \omega^2\mu\epsilon)E_y$

where → $\frac{d^2 E_y}{dx^2} = r^2 E_y$

$r = \alpha + j\beta$

$E_y = E_0 e^{-\alpha x} = E_0 e^{-\alpha x} e^{-j\beta x}$

$\frac{d^2 E_y}{dx^2} = j\omega\mu\sigma E_y = r^2 E_y$

$r = \frac{r^2}{\sqrt{j\omega\mu\sigma}} = \alpha + j\beta$

from $\sqrt{j} = \frac{1+1}{\sqrt{2}}$

we have; $r = \frac{\sqrt{\omega\mu\sigma}}{2} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

→ $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ & $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

∴ $E_y = E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}} \times e^{-\sqrt{\frac{\omega\mu\sigma}{2}}}$

∴ we have; → $E_y = E_0 e^{-x/\delta} e^{-jx/\delta}$

This Indicates that the amplitude of the wave decreases exponentially as it generated a conductive medium by a factor $\delta e^{-x/\delta}$

b) Skin depth is the depth of penetration of a wave inside a conductor

$\delta = \sqrt{2/\omega\mu\sigma}$ and $\omega = 2\pi f$

$= \sqrt{\frac{2}{2\pi f\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$

∴ $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$ or $\sqrt{\frac{1}{\pi f\mu\sigma}}$

$f = 10 MHz \approx 1 \times 10^7 Hz$

$\sigma = 5.8 \times 10^7$ S/m, $\mu_r = 1$, $\mu_0 = 1.257 \times 10^{-6}$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

$$\mu = \mu_r \times \mu_0 = 1 \times 1.257 \times 10^{-6}$$
$$1.257 \times 10^{-6}$$

$$\delta = \frac{1}{\sqrt{\pi \times 1 \times 10^7 \times 1.257 \times 10^{-6} \times 5.8 \times 10^{7}}}$$

$\delta = 2.09 \times 10^{-6}$ m

$b = 10$ mm $= 0.01$ m, $a = 3$ mm $= 0.003$, $\epsilon_0 = 8.85 \times 10^{-12}$, $\mu_0 = 1.257 \times 10^{-6}$

a Capacitance per meter

$$C = \frac{2\pi\epsilon_0}{\log b/a}$$

$$C = \frac{2\pi \times 8.85 \times 10^{-12}}{\log \frac{0.01}{0.03}}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\log 28.03}$$

$C = 2.84 \times 10^{-4}$ F/m

B

Inductance per meter

$$L = \frac{\mu_0}{2\pi} \log_e b/a$$

$$L = \frac{1.257 \times 10^{-6}}{2\pi} \log_e \frac{0.01}{0.03}$$

$$= \frac{1.257 \times 10^{-6}}{2\pi} \log 28.03$$

$L \simeq 2.90 \times 10^{-7}$ H/m

Charactaristics Impedance

$Z_0 = \sqrt{L/C}$

$Z_0 = \sqrt{\frac{2.90\times10^{-7}}{3.84\times10^{-4}}}$

$Z_0 = 86.90\,\Omega$

Phase velocity

$V_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.9\times10^{-7}\times3.84\times10^{-11}}} = 29.10\times10^{7}$