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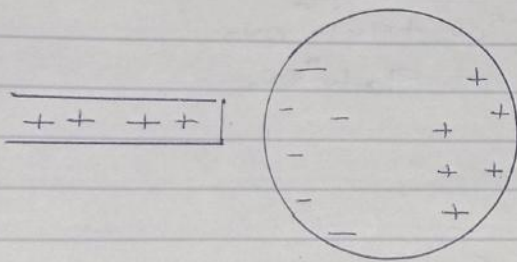
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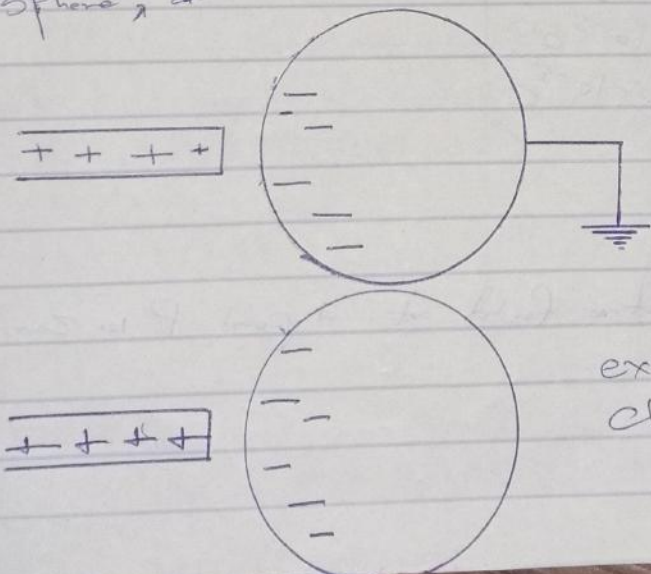
#### ④ Charging by Induction:

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction, whereby a positively charged rubber rod is brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere furthest away from the rod.



The region of the sphere nearest to the positively charged rod has an excess of negative charge because of the migration of protons away from this location.

If a grounded conducting wire is then connected to the sphere,



Some of the protons leave the sphere and travel to the earth. If the wire to ground is then removed, the conducting sphere is left with an excess of induced negative charge.



Finally, when the rubber rod is removed from the vicinity of the sphere, the sphere becomes uniformly distributed over the surface of the sphere.

1b)  $Q_1 + Q_2 = 5 \times 10^{-5} \text{ C}$

$F = 1 \text{ N}$

$d = 2 \text{ m}$

Calculate the charge on each sphere.

Sol.

$k = 9 \times 10^9$

$F = \frac{k Q_1 Q_2}{r^2}$

$1 = \frac{9 \times 10^9 \times (Q_1 Q_2 \cdot 5 \times 10^{-5})}{2^2}$

$4 = 9 \times 10^9 \times 5 \times 10^{-5} Q_1 + 9 \times 10^9 Q_2$

$4 = 4.5 \times 10^5 Q_1^2 + 9 \times 10^9 Q_2$

Quadratic eqn.

$9 \times 10^9 Q_2 - 4.5 \times 10^5 Q_1 + 4 = 0$

$Q_1 = 0.000011 \text{ C}$

$Q_2 = 0.000038 \text{ C}$

$\approx Q_1 = 1.11 \times 10^{-5} \text{ C}$

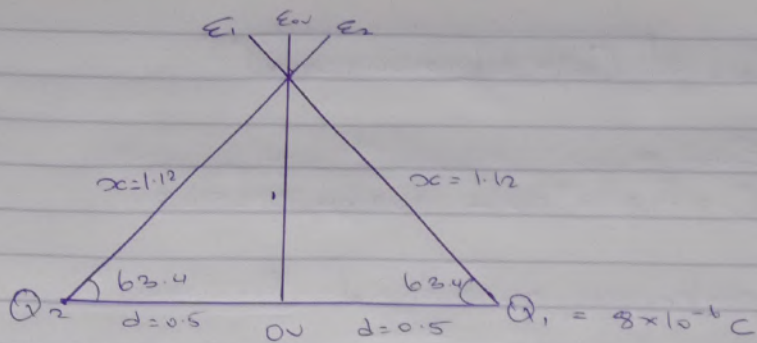
$\approx Q_2 = 3.8 \times 10^{-5} \text{ C}$

1c)  $Q_1 = Q_2 = 8 \mu\text{C}$

$d = 0.5 \text{ m}$

Determine if electric field at a point P is zero





$$\theta = \tan^{-1} \left( \frac{\text{opp}}{\text{adj}} \right)$$

$$\tan \theta = \frac{1}{0.5} = 2$$

$$\theta = \tan^{-1}(2)$$

$$\theta = 63.4$$

$$r =$$

$$r^2 = 1^2 + 0.5^2$$

$$\sqrt{r^2} = \sqrt{1.25}$$

$$r = \sqrt{1.25}$$

$$r = 1.12$$

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5739.7959$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = 5739.7959$$

$$E_{ov} = \frac{kQ_{ov}}{r^2} = \frac{9 \times 10^9 \times 0v}{1} = 9 \times 10^9 \text{ ov}$$

VECTOR	Angle	x-comp	y-comp
$E_1 = 5739.7959$	$63.4^\circ$	$E_1 \cos \theta$ $E_{1x} = -2570.045785$	$5132.262939$
$E_2 = 5739.7959$	$63.4^\circ$	$2570.045785$	$5132.262939$
$E_{ov} = 9 \times 10^9 \text{ ov}$	$90^\circ$	$E_{ov} \cos \theta = 0$ $E_{ox} = 0$	$9 \times 10^9 \text{ ov}$ $E_y = 10264.525878$

$$\text{Magnitude} = \sqrt{(E_{ox})^2 + (E_y)^2}$$

$$E_{\text{net}} = \sqrt{(0)^2 + (10264.52568)^2}$$

$$E_{\text{net}} = 0$$

$$0 = 9 \times 10^9 \text{ V} + 10264.52568$$

make  $0 \text{ V}$  subject of formulae.

$$0 \text{ V} = \frac{-10264.52568}{9 \times 10^9}$$

$$0 \text{ V} = 1.140502853 \times 10^{-6}$$

$$\underline{r = 11.44 \text{ cm}}$$

(3a) (i) Volume charge density,  $\rho = \frac{dQ}{dv} \rightarrow dQ = \rho dv$

(ii) ~~Surface charge density,  $\sigma = \frac{dQ}{dA}$~~

(iii) Surface charge density,  $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

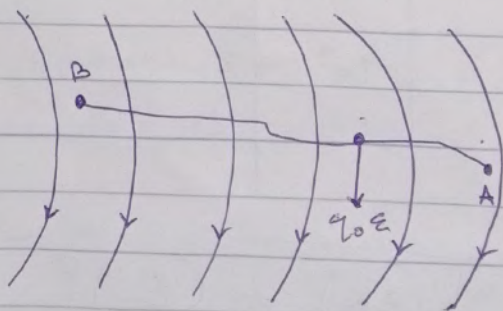
(iv) Linear charge density,  $\lambda = \frac{dQ}{dl} \rightarrow dQ = \lambda dl$

### (3b) ELECTRIC POTENTIAL DIFFERENCE.

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other.

It is measured in volt (V) or joules per coulomb (J/C).

It is a scalar quantity.





In the diagram above, suppose a test charge  $q_0$  is moved from point A to point B along an arbitrary path inside an electric field  $E$ . The electric field  $E$  exerts a force  $F = q_0 E$  on the charge as shown in the figure above.

To move the test charge from A to B at constant velocity, an external force of  $F = -q_0 E$  must act on the charge. Therefore, the elemental work done  $dW$  is given as:

$$dW = F \cdot dl \quad \text{--- (i)}$$

But:

$$F = -q_0 E \quad \text{--- (ii)}$$

Subst eqn (ii) in (i)

$$dW = -q_0 E \cdot dl \quad \text{--- (iii)}$$

Then total work done moving the test charge from A to B

$$W(A \rightarrow B)_{Ag} = -q_0 \int_A^B E \cdot dl \quad \text{--- (iv)}$$

From the definition of electric potential difference, it follows that

$$V_B - V_A = \frac{W(A \rightarrow B)_{Ag}}{q_0} \quad \text{--- (v)}$$

$$V_B - V_A = - \int_A^B E \cdot dl \quad \text{--- (vi)}$$

(4) Magnetic flux is defined as the strength of the magnetic field, which can be represented by lines of force. It is represented by the symbol  $\Phi$ .  
mathematically given as  $\Phi = B \cdot dA$

(b)  $H = 9 \times 10^{-31} \text{ kg}$   
 $r = 1.4 \times 10^{-7} \text{ m}$

$B = 3.5 \times 10^{-1} \text{ meters}^2$

Cyclotron frequency = Angular Speed.

$$\omega = \frac{v}{r} = \frac{qvB}{m}$$

$$\omega = \frac{qvB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.1 \times 10^{-31}}$$

~~$$\omega = 6222222222.2222 \text{ T}^{-1}$$~~

~~$$\omega = 6.2 \times 10^{10}$$~~

$$\omega = 6222222222.2222 \text{ T}^{-1}$$

- 4c We were given parameters such as
- Mass of the electron =  $9.11 \times 10^{-31} \text{ kg}$
  - A radius of  $1.4 \times 10^{-7} \text{ m}$ .
  - Magnetic field of  $3.5 \times 10^{-1} \text{ meter square}$ .

Asked to find frequency the cyclotron frequency which is equal or the same thing as angular speed. It is called cyclotron frequency because it is a frequency in an accelerator called cyclotron.

Recall that angular speed is given as  $\omega = \frac{v}{r} = \frac{qv}{m}$

Substituting we have.

$$\omega = \frac{v}{r} = \frac{qv}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$= 6222222222.2222 \text{ T}^{-1}$$

So since cyclotron frequency is equal to angular speed the cyclotron frequency is equal to  $6222222222.2222 \text{ T}^{-1}$ , having a unit as  $1/\text{s}$  which is equal to the unit of frequency dimensionally.



(5a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space ( $\mu_0$ ), the current ( $I$ ), the change in length, the radius and inversely proportional to square of radius ( $r^2$ ). It can be represented mathematically by-

$$\vec{dB} = \frac{\mu_0 I d\vec{l} \times \vec{r}}{4\pi r^2}$$

Where  $\mu_0$  is a constant called permeability of free space.

$$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

The unit of  $B$  is meter square.

(5b) Magnetic field of a straight current carrying conductor

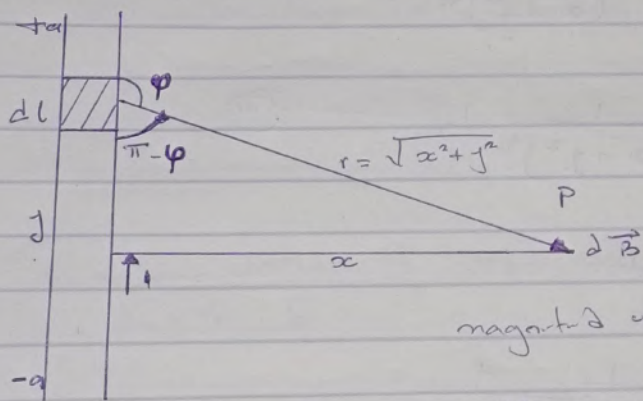


Diagram showing a section of a straight current carrying conductor.

Applying the Biot-Savart law, we find the magnitude of the field  $\vec{dB}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

From diagram,  $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \text{--- (1)}$$

$$\text{But } \sin(\pi - \phi) = \frac{xc}{\sqrt{x^2 + y^2}} = \frac{xc}{(x^2 + y^2)^{1/2}} \quad \text{--- (2)}$$

Substituting (ii) in (i)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall  $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \text{--- (iii)}$$

Using special integrals.

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

Eqn (iii) becomes:

$$B = \frac{\mu_0 I x}{4\pi} \left[ \frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left[ \frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right]$$

$$B = \frac{\mu_0 I a^2}{2\pi x^2 (x^2 + a^2)^{1/2}}$$



$$B = \frac{\mu_0 I}{4\pi x} \left[ \frac{2a}{(x^2 + a^2)^{1/2}} \right]$$

When the length  $2a$  of the conductor is very great in comparison to its distance  $x$  from point  $P$ , we consider it infinitely long. That is, when  $a$  is much larger than  $x$ .

$$(x^2 + a^2)^{1/2} \cong a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the  $J$ -axis. Thus, at all points in a circle of radius  $r$ , around the conductor, the magnitude of  $B$  is

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{--- (10)}$$

Eqn (10) defines the magnitude of the magnetic field or flux density  $B$  near a long, straight current carrying conductor.