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from $\frac{d^2E_y}{dx^2} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)E_y$

where $\rightarrow \frac{d^2E_y}{dx^2} = \gamma^2 E_y$

$\gamma = \alpha + j\beta$

$E_y = E_0 e^{-\gamma x} = E_0 e^{-\alpha x} e^{-j\beta x}$

$\frac{d^2E_y}{dx^2} = j\omega\mu\sigma E_y = \gamma^2 E_y$

$\gamma = \frac{\gamma^2}{\sqrt{j\omega\mu\sigma}} = \alpha + j\beta$

from $\sqrt{j} = \frac{1+1}{\sqrt{2}}$

We have; $\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

$\rightarrow \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ & $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$\therefore E_y = E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x}$

$\therefore$ ~~we have~~ we have $j \longrightarrow E_y = E_0 e^{-x/\delta} e^{-jx/\delta}$

This indicates that the amplitude of the wave decreases exponentially as it generates a conductive medium by a factor of $e^{-x/\delta}$

b Skin depth is the depth of penetration of a wave inside a conductor

$$\delta = \sqrt{2/\omega\mu\sigma} \quad \text{and} \quad \omega = 2\pi f$$

$$= \sqrt{\frac{2}{2\pi f \mu r}} = \sqrt{\frac{1}{\pi f \mu r}}$$

$$\therefore \delta = \sqrt{\frac{2}{\omega \mu r}} \quad \text{or} \quad \sqrt{\frac{1}{\pi f \mu r}}$$

c $f = 10MHz \simeq 1 \times 10^{7} Hz$

$\sigma = 5.8 \times 10^{7} \,℧/m$, $\mu_r = 1$, $\mu_0 = 1.257 \times 10^{-6}$

$$\delta = \sqrt{\frac{1}{\pi f \mu r}} = \frac{1}{\sqrt{\pi f \mu r}}$$

$$\mu = \mu_r \times \mu_0 = 1 \times 1.257 \times 10^{-6}$$

$$1.257 \times 10^{-6}$$

$$\delta = \frac{1}{\sqrt{\pi \times 1 \times 10^{7} \times 1.257 \times 10^{-6} \times 5.8 \times 10^{7}}}$$

$$\delta = 2.09 \times 10^{-5} \,m$$

$b = 10mm$, $a = 3mm$, $\epsilon_0 = 8.85 \times 10^{-12}$

$= 0.01m$, $= 0.003$, $\mu_0 = 1.257 \times 10^{-6}$

a Capacitance per meter

$$C = \frac{2\pi \epsilon_0}{\log b/a}$$

$$C = \frac{2\pi \times 8.85 \times 10^{-12}}{\log_e \frac{0.01}{0.03}}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\log 28.03}$$

$$C = 3.84 \times 10^{-4} F/m$$

b

Inductance per meter is

$$L = \frac{\mu_0}{2\pi} \log_e b/a$$

$$L = \frac{1.057 \times 10^{-6}}{2\pi} \cdot \log_e \frac{0.01}{0.03}$$

$$= \frac{1.257 \times 10^{-6}}{2\pi} \log 28.03$$

$$L = 2.90 \times 10^{-7} H/m$$

Characteristics Impedance

$$Z_0 = \sqrt{L/C}$$

$$Z_0 = \sqrt{\frac{2.90 \times 10^{-7}}{3.84 \times 10^{-4}}} \qquad Z_0 = 86.90 \Omega$$

Phase velocity

$$V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.9 \times 10^{-7} \times 3.84 \times 10^{-11}}} = 29.10 \times 10^{7} V$$