

4a Magnetic flux is defined as the strength of a magnet represented by lines of force. Its symbol (it can be represented by ϕ)

4b Charge, $q = -1.6 \times 10^{-19} \text{ C}$ $\theta = 90$
 $B = 3.5 \times 10^{-1} \text{ weber/meter square}$ $\sin 90 = 1$
 $r = 1.4 \times 10^{-7} \text{ m}$
 $m_{\text{electron}} = 9.11 \times 10^{-31} \text{ kg}$

Cyclotron frequency, $\omega = \frac{qB}{m}$

$$= \frac{-1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$= -6.15 \times 10^{10} \text{ rad/s}$$

4c The Particle of mass $9.11 \times 10^{-31} \text{ kg}$ is moving in a circle because its force is perpendicular to the velocity and

4c A/ The Particle with mass of $9.11 \times 10^{-31} \text{ kg}$ with charge $-1.6 \times 10^{-19} \text{ C}$ which is perpendicular to the magnetic field $3.5 \times 10^{-1} \text{ Tesla}$ will move in a circular motion with a cyclotron frequency of $6.15 \times 10^{10} \text{ rad/s}$

5

5a The Biot - Savart law is based on the following observation for magnetic field $d\vec{B}$ at point P, length $d\vec{l}$ of a wire and steady current \vec{I} .

i) The vector $d\vec{B}$ is perpendicular both to $d\vec{l}$ (which points in the direction of the current) and to the unit vector \hat{r} directed from $d\vec{l}$ toward P

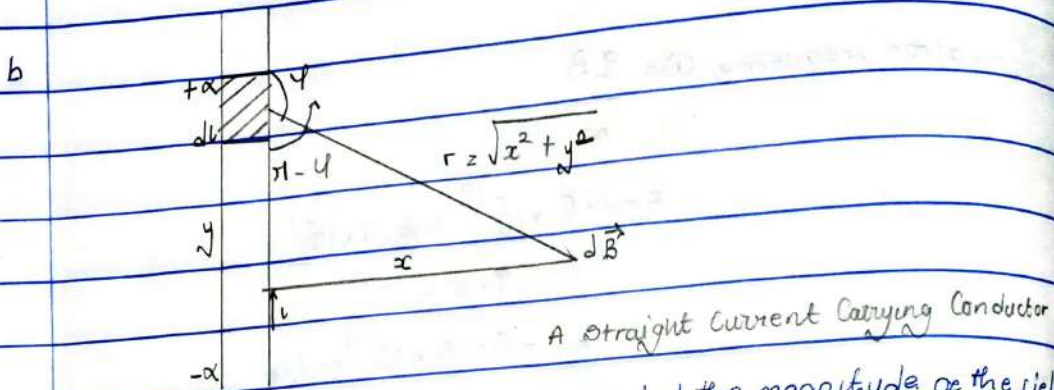
i The magnitude of $d\vec{B}$ is inversely proportional to r^2 , where r is the distance from $d\vec{l}$ to P .

ii The magnitude of $d\vec{B}$ is proportional to the current I and to the magnitude of the length element $d\vec{l}$.

iii The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between \hat{r} and $d\vec{l}$.

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$



Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin\phi}{r^2}$$

$$\sin(\pi - \phi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

From the diagram $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \dots (1)$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad (2)$$

Substituting (2) in (1)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \cdot x}{(x^2 + y^2)^{1/2} (y^2 + x^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dx}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \dots (u)$$

Using special integrals

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

equation (u) becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{x^2 + a^2} \right)$$

When the length $2a$ of the conductor is very great in comparison to the distance x from B , it is considered infinitely long. That is, when a is much larger than x .

$$(x^2 + a^2)^{1/2} \approx a, \text{ as } a \rightarrow \infty$$

$$B = \frac{\mu_0 I}{2\pi x}$$

2a Electric Field

This is a region of space in which an electric charge will experience an electric force.

Electric field intensity
This is the force per unit charge

b $Q_1 = 8 \mu\text{C} = 8 \times 10^{-9} \text{C}$

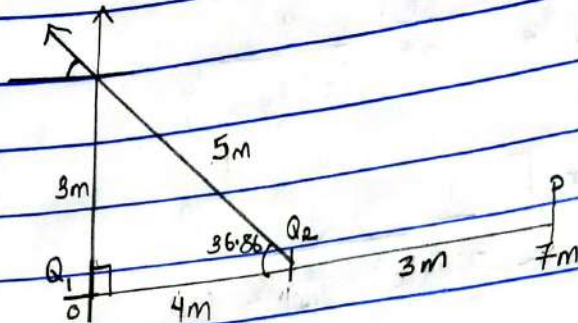
$Q_2 = 12 \text{ nC} = 12 \times 10^{-9}$

$x_1 = 4 \text{ cm}$

$\cos \theta = \frac{4}{5}$

5

$\theta = 36.86^\circ$



1 The net electric field at point P on the x-axis at $x = 7 \text{ cm}$

$$E_1 = \frac{8 \times 10^{-9} \times 9 \times 10^9}{7^2}$$

$$= \frac{72}{49} = 1.459 \text{ N/C} \approx 1.46$$

$$E_2 = \frac{12 \times 10^{-9} \times 9 \times 10^9}{3^2}$$

$$= 12 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2 = 12 + 1.46$$

$$= 13.46 \text{ N/C}$$

$$= 13.5 \text{ N/C}$$

2 $E_1 = \frac{8 \times 10^{-9} \times 9 \times 10^9}{3^2} = 8 \times 1 = 8 \text{ N/C}$

$$E_2 = \frac{12 \times 10^{-9} \times 9 \times 10^9}{5^2}$$

$$= 4.32 \text{ N/C}$$

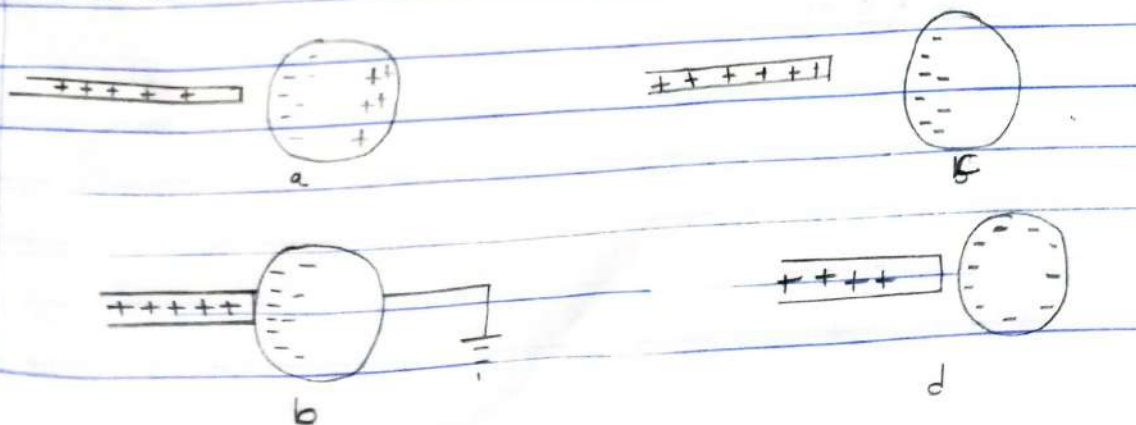
E	Angle	x-component	y-component
8 N/C	90°	$E_{x1} = 8 \times \cos 90$ $= 0 \text{ N/C}$	$E_{y1} = 8 \times \sin 90$ $= 8 \text{ N/C}$
4.32 N/C	36.86°	$E_{x2} = 4.32 \times \cos 36.86$ $= -3.45 \text{ N/C}$	$E_{y2} = 4.32 \times \sin 36.86$ $= 2.58 \text{ N/C}$
Total		$E_x = -3.45 \text{ N/C}$	$E_y = 10.58 \text{ N/C}$

$$E = \sqrt{(-3.45)^2 + (10.58)^2}$$

$$= \sqrt{123.8389}$$

$$E = 11.13 \text{ N/C}$$

1a. Bring a positively charged rubber rod to a neutral conducting sphere that is isolated. The repulsive force between electrons in the rod and those in the sphere cause redistribution of charges in the sphere. The region close to the sphere would have excess negative charge, cause the positive charges would have moved from this location. Connect the sphere to a grounded conducting wire, some electrons leave the sphere and into the earth. Remove the wire and sphere is left with excess of induced negative charge, which later when the rod is removed become uniformly distributed in the sphere.



16

$$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$$

$$q_1 = 5 \times 10^{-5} \text{ C} - q_2$$

$$F = 1 \text{ N}$$

$$r = 2 \text{ m}$$

$$k = 9 \times 10^9$$

$$F = \frac{k Q_1 Q_2}{r^2}$$

$$Q_1 Q_2 = \frac{F r^2}{k} = \frac{1 \times 4}{9 \times 10^9} = 4.44 \times 10^{-10}$$

$$4.44 \times 10^{-10} = q_1 \times (5 \times 10^{-5} - q_1)$$

$$5 \times 10^{-5} q_1 - q_1^2 = 4.44 \times 10^{-10}$$

$$q_1^2 - 5 \times 10^{-5} q_1 + 4.44 \times 10^{-10} = 0$$

Using algebraic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q_1 = \frac{5 \times 10^{-5} \pm \sqrt{(5 \times 10^{-5})^2 - 4(1)(4.44 \times 10^{-10})}}{2}$$

$$\text{or } q_1 = \frac{5 \times 10^{-5} \mp \sqrt{(5 \times 10^{-5})^2 - 4(1)(4.44 \times 10^{-10})}}{2}$$

$$q_1 = 3.84 \times 10^{-5} \quad \text{or } 1.15 \times 10^{-5}$$

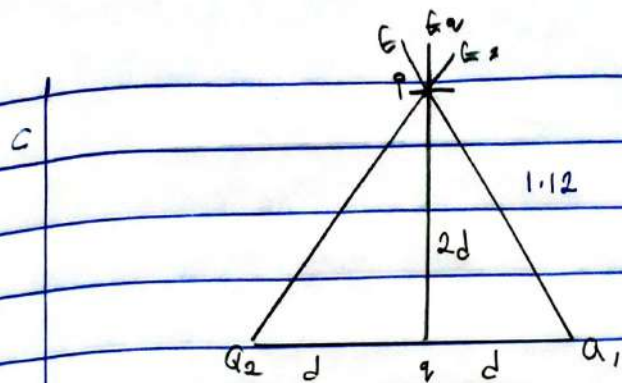
$$q_2 = 5 \times 10^{-5} - 1.15 \times 10^{-5}$$

$$= 3.85 \times 10^{-5}$$

$$q_1 = 1.15 \times 10^{-5}$$

$$q_2 = 3.85 \times 10^{-5}$$

} or vice-versa.



$$d = 0.5$$

$$k = 9 \times 10^9$$

$$Q_1 = Q_2 = 8 \mu\text{C}$$

$$\tan \theta = \frac{2d}{2d} = 1$$

$$\theta = \tan^{-1}\left(\frac{1}{0.5}\right) = 63.43^\circ$$

$$k_2 = \frac{kq_2}{r} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 57397.95918 \text{ N/C}$$

$$G_1 = 57397.95918 \text{ N/C}$$

$$E_q = \frac{kq}{r^2} = \frac{9 \times 10^9 \times q}{1} = 9 \times 10^9 q$$

Vector	Angle	x-component	y-component
$G_1 = 57397.95918$	63.43	-25673.58186	$+51336.07808$
$G_2 = 57397.95918$	63.43	$+25673.58186$	$+51336.07808$
Total =		$k_x = 0$	$k_y = 102672.1562$

$$E_q = \sqrt{0^2 + (102672.1562)^2}$$

$$E_q = 102672.1562$$

$$E_q = \frac{kq}{r} = 9 \times 10^9 q$$

$$q = \frac{102672.1562}{9 \times 10^9}$$

$$= 1.14 \times 10^{-5}$$

$$= 11.4 \times 10^{-6} \text{ C} = \cancel{11.4 \mu\text{C}} \quad 11.4 \mu\text{C}$$