

SECTION A

Electric field and electric field intensity

Electric field

⇒ It is a region of space in which any electric charge will experience an electric force

Electric field intensity

It is the force per unit charge

2 $q_1 = 8 \text{ nC}$ at origin. $q_2 = 12 \text{ nC}$ on x axis at $x = 4 \text{ m}$

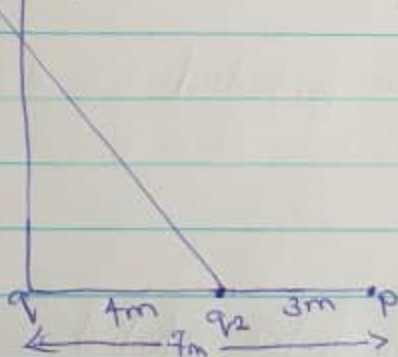
i) net electric field at point P on the axis at $x = 7 \text{ m}$

ii) Electric field at a point Q on the y axis at $y = 3 \text{ m}$ due to the charges.

Solutions

i)

$E_2 \uparrow$
 $E_1 \uparrow$



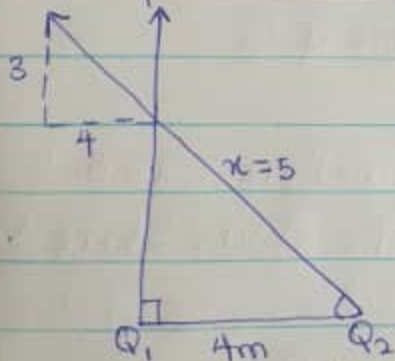
$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2}$$

$$= 1.469 \text{ N/C} \approx 1.5 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{7^2} = 12 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2 = 12 + 1.5 = 13.5 \text{ N/C}$$

ii) E at point Q on the y axis at $y = 3 \text{ m}$ due to charge



$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 3^2 = 5$$

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

Vector	Angle	x-comp	y-comp
$F_1 = 8\text{N/c}$	90°	0N/c	8N/c
$F_2 = 4.32\text{N/c}$	36.87°	-3.45N/c	2.59N/c
		$E_{fx} = -3.45\text{N/c}$	$E_{fy} = 10.59\text{N/c}$

$$E_{\text{net}} = \sqrt{E_{fx}^2 + E_{fy}^2}$$

$$E_{\text{net}} = 11.12\text{N/c}$$

3) Formulation of identities of charge

- i) Volume charge density $\rho = dQ/dv = dQ = \rho dv$
- ii) Surface charge density $\sigma = dQ/dA = dQ = \sigma dA$
- iii) Inner charge density $\lambda = dQ/dk = dQ = \lambda dk$

b) electric potential difference equation

- due to a single point charge

$$V_B - V_A = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

where $Q =$ point charge $v =$ electric potential

$r_B =$ distance of Q to point B

$r_A =$ distance of Q to point A

- due to several point charges

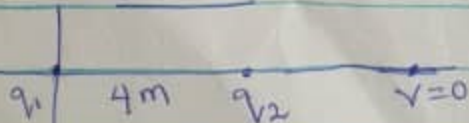
$$V_p = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right] \text{ where } v = \text{electric potential}$$

$Q =$ point charge

$r =$ distance of Q

3c) Point charge $Q_1 = 10\mu\text{C}$ $Q_2 = -2\mu\text{C}$ along x-axis $x=0$
 $x=4\text{m}$ respectively. find the position along the x-axis where $v=0$

Solution



$$V_p = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right]$$

$$\Rightarrow \text{Recall } \frac{1}{4\pi\epsilon_0} = k = 9 \times 10^9$$

$$V_p = k \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right]$$

$$V_p = 9 \times 10^9 \times \left[\frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$V_p = 0 = 9 \times 10^9 \times \left[\frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$0 = \frac{10 \times 10^{-6}}{4+x} + \frac{-2 \times 10^{-6}}{x} = \frac{10 \times 10^{-6}}{4+x} = \frac{2 \times 10^{-6}}{x}$$

$$10 \times 10^{-6} x = (4+x) (2 \times 10^{-6})$$

$$10 \times 10^{-6} x = 8 \times 10^{-6} + 2 \times 10^{-6} x$$

$$8 \times 10^{-6} = 10 \times 10^{-6} x - 2 \times 10^{-6} x$$

$$8 \times 10^{-6} = 8 \times 10^{-6} x$$

$$x = \frac{8 \times 10^{-6}}{8 \times 10^{-6}} = 1$$

\therefore position along the x-axis is 1m

where $v = 0$

$$v = k \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2} \right]$$

$$0 = \left[\frac{10 \times 10^{-6}}{4-x} + \frac{-2 \times 10^{-6}}{x} \right]$$

$$\frac{2 \times 10^{-6}}{x} = \frac{10 \times 10^{-6}}{4-x}$$

$$(4-x) (2 \times 10^{-6}) = 10 \times 10^{-6} x$$

$$8 \times 10^{-6} - 2 \times 10^{-6} x = 10 \times 10^{-6} x$$

$$8 \times 10^{-6} = 10 \times 10^{-6} x + 2 \times 10^{-6} x$$

$$8 \times 10^{-6} = 12 \times 10^{-6} x$$

$$x = \frac{8 \times 10^{-6}}{12 \times 10^{-6}} = 0.67 \text{ m}$$

\therefore position of $v = 0$ is 0.67m

SECTION B

4a) Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is denoted as ϕ .

$$\phi = B \cdot dA$$

b) $m_e = 9.11 \times 10^{-31} \text{ kg}$, $r = 1.4 \times 10^{-7} \text{ m}$, $B = 3.5 \times 10^{-1} \text{ W/m}^2$

cyclotron frequency = angular speed $q = 1.6 \times 10^{-19}$

$$F_B = qvB = \frac{m_e v^2}{r}$$

$$m_e v = qBr$$

$$v = \frac{qBr}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1} \times 1.4 \times 10^{-7}}{9.11 \times 10^{-31}} = 8.6 \times 10^{-3} \text{ m/s}$$

$$\omega = \frac{v}{r} = \frac{qB}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = 6.14 \times 10^{10} \text{ s}^{-1}$$

4c) In 4b we were given parameters; mass of electron = $9.11 \times 10^{-31} \text{ kg}$
radius = $1.4 \times 10^{-7} \text{ m}$ $B = 3.5 \times 10^{-1} \text{ W/m}^2$

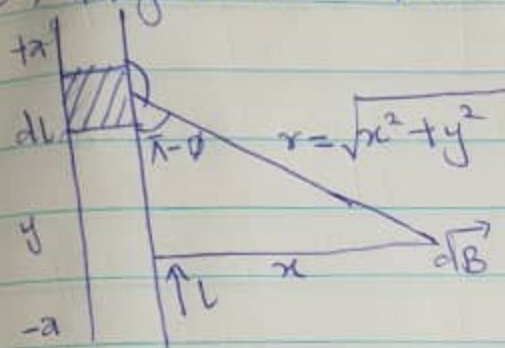
We were asked to find the cyclotron frequency which is the same thing as angular speed. It is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall $\omega = \text{angular speed}$; $\omega = \frac{qB}{m_e}$ since cyclotron frequency = angular speed

The cyclotron frequency = $6.14 \times 10^{10} \text{ s}^{-1}$ having a unit of $\frac{1}{T}$ which is the unit of frequency dimensionally.

5a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to the square of radius. Mathematically, $d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$

5b) Magnetic field of a straight current carrying conductor



A section of a straight current carrying conductor.

Applying Bio-savart law, we find the magnitude of the field ($d\vec{B}$) from the diagram.

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \phi$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

from the diagram $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \dots (i)$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \dots (ii)$$

substitute (ii) into (i)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$dl = dy : B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \dots (iii)$$

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right) ; (x^2 + a^2)^{1/2} = a = \infty$$

$$B = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 I}{2\pi r}$$