

When the length  $2a$  of the conductor is very great in comparison to its distance  $x$  from point  $P$ , we consider it infinitely long. Let  $B$ , when  $a$  is much larger than  $x$ ,  $(x^2 + a^2)^{1/2} \approx a + \frac{x^2}{2a} \approx a$

$$\therefore B = \frac{\mu_0 I}{2a x}$$

In a physical situation we have equal symmetry about the  $y$ -axis is thus at all points in a circle of radius  $a$ , around the conductor the magnetic field at  $P$  is:  $B = \frac{\mu_0 I}{2a x}$

Applying the Biot-Savart law we find the magnitude of the field

$$B = \frac{\mu_0 I}{4a} \int_{-a}^a \frac{d \sin \phi}{r^2}$$

$$B = \frac{\mu_0 I}{4a} \int_{-a}^a \frac{dl \sin(\alpha - \phi)}{r}$$

from diagram  $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4a} \int_{-a}^a \frac{dl \sin(\alpha - \phi)}{x^2 + y^2} \quad \dots (i)$$

$$\text{But } \sin(\alpha - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \dots (ii)$$

$$\text{Sub (ii) into (i)} \quad B = \frac{\mu_0 I}{4a} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{4a} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall  $dl = dy$

$$B = \frac{\mu_0 I}{4a} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4a} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \dots (iii)$$

Using special integral:  $\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$

$$\text{Eqn (iii) becomes } B = \frac{\mu_0 I x}{4a} \left[ \frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$= \frac{\mu_0 I x}{4a} \left( \frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$= \frac{\mu_0 I}{2a x} \left( \frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

$$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$= 6.22 \times 10^{10} \text{ T}^{-1}$$

Mass of electron =  $9.11 \times 10^{-31} \text{ kg}$   
 radius =  $1.4 \times 10^{-7}$   
 magnetic field =  $3.5 \times 10^{-1}$

Cyclotron frequency can be called Angular speed  
 i.e. that Angular  $\omega = \frac{v}{r} = \frac{qB}{m}$

substituting we have  $\omega = \frac{qB}{m}$

$$= \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$= 6.22 \times 10^{10} \text{ T}^{-1}$$

So cyclotron frequency =  $6.22 \times 10^{10} \text{ T}^{-1}$ , the unit is equal to unit of frequency dimensionally.

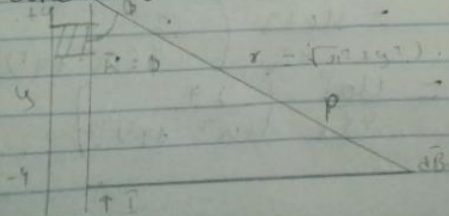
1) Biot - Savart law states that the magnetic field is directly proportional to the product permeability of free space ( $\mu_0$ ), the current ( $I$ ), the change in length the radius and inversely proportional to the square of radius ( $r^2$ ) It can be represented mathematically

$$dB = \frac{\mu_0 I dl \times r}{4\pi r^3} \text{ where } \mu_0 \text{ is constant}$$

called permeability of free space

$$= 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

2) Magnetic field of a straight current carrying conductor



### 3b) Electric potential difference

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to another. It is measured in Volt (V) or Joules per coulomb (J/C).

It is a scalar quantity.

Elemental work done  $dW$  is given as

$$dW = F \cdot dl \quad \text{--- (i)}$$

$$F = q_0 E \quad \text{--- (ii)}$$

$$\text{Substitute equation (ii) in (i) } \Rightarrow dW = -q_0 E dl \quad \text{--- (iii)}$$

Total work done in moving the test charge from A to B is

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E dl \quad \text{--- (iv)}$$

From the definition of electric potential difference follows that:

$$V_B - V_A = \frac{W(A \rightarrow B)_{q_0}}{q_0} \quad \text{--- (v)}$$

Putting equation 4 in 5 yields

$$V_B - V_A = \int_A^B E dl \quad \text{--- (vi)}$$

### Section B

4a.) Magnetic flux can be defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol  $\Phi$ .

$$4.b) m = 9 \times 10^{-31} \text{ kg}$$

$$r = 1.4 \times 10^{-7} \text{ m}$$

$$B = 3.5 \times 10^{-1}$$

Cyclotron frequency = Angular speed

$$\omega = v/r = qB/m$$

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-4}}{(1.12)^2} = 57397.9598$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 18 \times 10^{-6}}{(1.12)^2} = 57397.95918$$

$$E_3 = \frac{kq_3}{r^2} = \frac{9 \times 10^9 \times 9 \times 10^{-9}}{1} = 9 \times 10^9 \text{ e}$$

Vectors	Angle	x-component	y-component
$E_1 = 57397.95918$	$63.4^\circ$	$E_1 \cos \theta = 2570.045785$	$E_1 \sin \theta = 5132.262839$
$E_2 = 57397.95918$	$63.4^\circ$	$2570.045785$	$5132.262839$
$E_3 = 9 \times 10^9 \text{ e}$	$90^\circ$	$E_3 \cos \theta = 0$	$9 \times 10^9 \text{ e}$
		$\Sigma x = 0$	$\Sigma y = 10264.52568$

$$\text{Magnitude} = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E_y = \sqrt{0^2 + (10264.52568)^2}$$

$$\text{Since } E_x = 0$$

$$0 = 9 \times 10^9 \text{ e} + 10264.52568$$

Making  $q$  subject of formula

$$q = \frac{10264.52568}{9 \times 10^9}$$

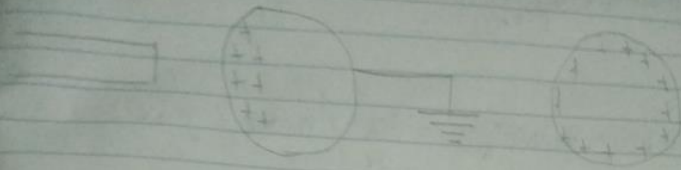
$$= 1.140502853 \times 10^{-16}$$

$$= 11.4 \mu\text{C}$$

i) Volume charge density  $\rho = \frac{dQ}{dV}$  'n'  $dQ = \rho dV$

ii) Surface charge density  $\sigma = \frac{dQ}{dA}$  'n'  $dQ = \sigma dA$

iii) Linear charge density  $\lambda = \frac{dQ}{dl}$  'n'  $dQ = \lambda dl$



b)  $k = 9 \times 10^9$   
 $q_1 + q_2 = 5 \times 10^{-5}$   
 $\phi = 1 \text{ N}$   
 $d = 2 \text{ m}$

(charge on each sphere = ?)

$$\phi = \frac{k q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1 q_2 \times 5 \times 10^{-5})}{2^2}$$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2$$

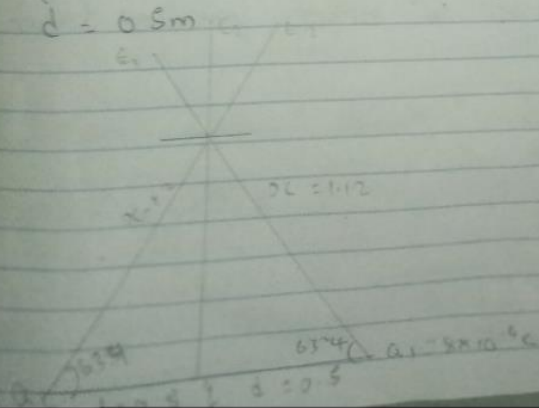
$$9 \times 10^9 q_2 - 4.5 \times 10^5 q_1 + 4 = 0$$

$$q_1 = 0.0000111 \text{ C} \approx 1.11 \times 10^{-5} \text{ C}$$

$$q_2 = 0.0000389 \text{ C} \approx 3.89 \times 10^{-5} \text{ C}$$

c)  $Q_1 = Q_2 = 8 \mu\text{C}$

$d = 0.5 \text{ m}$



$$r^2 = 1^2 + 0.5^2$$

$$\sqrt{r^2} = \sqrt{1.25}$$

$$r = \sqrt{1.25}$$

$$r = 1.12$$

$$\tan \theta = \frac{\text{OPP}}{\text{ADJ}}$$

$$= \frac{0.5}{1}$$

$$\theta = \tan^{-1}(0.5)$$

$$\theta = 63.4^\circ$$

Alpokinlavo Eselobare  
Computer Science  
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PHY103

### Section A

(a.) Static charges can be obtained on an object without touching it by a process called electrostatic induction. Consider a negatively charged rubber rod brought near a charged conducting sphere that is insulated so that there is no conduction path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere nearest the rod (Fig. 1.3a). The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from its location. A grounded conducting wire is then connected to the sphere as in (Fig. 1.3b). Some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed (Fig. 1.3c) the conducting sphere is left with an excess of induced positive charge. Finally when the rubber rod is removed from the vicinity of the sphere (Fig. 1.3d) the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

