IGWE KAMSOCHI NGOZI-CHIBUZOR

19/MHS01/196

MEDICINE AND SURGERY

PHY102 ASSIGNMENT

5. State Biot-Savart Law.

Biot-Savart Law states that the magnetic intensity dB at a point A due to current I flowing through a small element dl is

i. Directly proportional to current (I)

ii. Directly proportional to the length of the element (dl)

iii. Directly proportional to the sine of angle θ between the direction of current and the line joining the element dl from point A.

iv. Inversely proportional to the square of the distance (r) of point A from element dl.

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

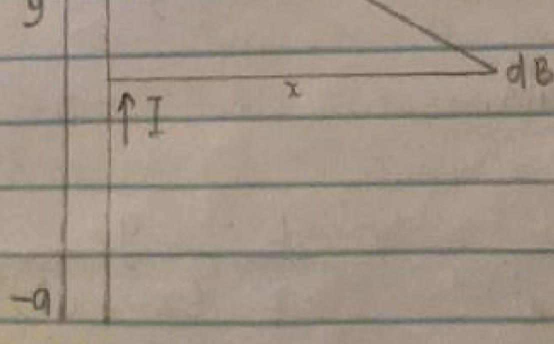
$$dB = k\,\frac{I\,dl \sin\theta}{r^2}$$

$$\text{where } k = \frac{\mu_0 \mu_r}{4\pi}$$

μ_0 = Absolute permittivity of air in vacuum

μ_r = relative permittivity

b



4. Magnetic flux is defined as the number of magnetic field lines passing through a given closed surface. It is a measurement of the total magnetic field which passes through a given area. Magnetic flux through a surface is the surface integral of the normal component of the magnetic field (B) passing through that surface. It is denoted by Φ or Φ_B. The CGS unit is the maxwell and the SI unit of magnetic flux is the weber (Wb).

Magnetic flux formula is given by :

$$\Phi_B = B \cdot A = BA \cos\theta$$

where Φ_B - magnetic flux
B - magnetic field
A - Area
θ - angle at which the field lines pass through the given surface area.

b cyclotron frequency, $f = \frac{qB}{2\pi m}$

$$f = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{2\pi(9.11 \times 10^{-31})}$$

$$f = \frac{5.6 \times 10^{-20}}{5.72 \times 10^{-30}}$$

$$f = 9.79 \times 10^{9}\ Hz$$

$$f = 9.79\ GHz$$

c. The formula for cyclotron frequency, f is given as

$f = \frac{qB}{2\pi m}$ where q - charge
B - magnetic field
m - mass of electron

Cyclotron frequency is independent of the radius and velocity and therefore independent of the particle's kinetic energy.

$$0 = 9\times10^{9} \times \left[\frac{10\times10^{-6}}{x} - \frac{2.0\times10^{-6}}{y}\right]$$

$$0 = 9\times10^{9} \times \left[\frac{10\times10^{-6}y - 2.0\times10^{-6}x}{xy}\right]$$

Cross multiply

$$0 \times xy = 9\times10^{9} \times [10\times10^{-6}y - 2.0\times10^{-6}x]$$

$$0 = 9\times10^{9} \times [10\times10^{-6}y - 2.0\times10^{-6}x]$$

Divide through by 9×10^{9}

$$\frac{0}{9\times10^{9}} = \frac{9\times10^{9} \times [10\times10^{-6}y - 2.0\times10^{-6}x]}{9\times10^{9}}$$

$$0 = 10\times10^{-6}y - 2.0\times10^{-6}x$$

$$2.0\times10^{-6}x = 10\times10^{-6}y$$

Divide through by 2.0×10^{-6}

$$\frac{2.0\times10^{-6}x}{2.0\times10^{-6}} = \frac{10\times10^{-6}y}{2.0\times10^{-6}}$$

$$x = 5y$$

$\longrightarrow$ $y = x - 4$

$$\therefore x = 5(x-4)$$

$$x = 5x - 20$$

$$x - 5x = -20$$

$$-4x = -20$$

$$x = \frac{-20}{-4} = 5m$$

At point, 5m on the x-axis is when $V = 0$.

3. Volume charge density ; $\rho = \frac{q}{V}$, where q - charge
SI unit : m^{-3} V - volume of distribution

ii Surface charge density; $\sigma = \frac{q}{A}$, where q - charge
SI unit : m^{-2} A - Area of surface

iii Linear charge density ; $\lambda = \frac{q}{L}$, where q - charge
SI unit : m^{-1} L - Length over which the charge is distributed.

b. Electrical potential difference

Electrical potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in volts (v) or Joules per Coulomb ($J \cdot C^{-1}$). It is a scalar quantity.

$$V_B - V_A = \frac{W(A \rightarrow B)_{Ag}}{q_0}$$

Recall $W(A \rightarrow B)_{Ag} = -q_0 \int_A^B E\,dL$

$$V_B - V_A = -\int_A^B E\,dL$$

c. $q_1 = 10\,\mu C$
$q_2 = -2\,\mu C$
$r_1 = x$
$r_2 = x - 4 = y$
where $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

$$\longrightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

1c continued

$$E_q = \sqrt{E_x^2 + E_y^2}$$

$$= \sqrt{(0)^2 + [(1.026 \times 10^5) + 9.0 \times 10^9 q]^2}$$

$$E_q = \sqrt{[(1.026 \times 10^5) + 9.0 \times 10^9 q]^2}$$

But $E_q = 0$

$$0 = \sqrt{[(1.026 \times 10^5) + 9.0 \times 10^9 q]^2}$$

$$\left.\begin{aligned} 0 &= \sqrt{(1.026 \times 10^5)^2 + (9.0 \times 10^9 q)^2} \\ 0 &= \sqrt{1.053 \times 10^{10} + (9.0 \times 10^9 q)^2} \end{aligned}\right\} \text{IRRELEVANT}$$

$$0 = 1.026 \times 10^5 + 9.0 \times 10^9 q$$

$$9.0 \times 10^9 q = -1.026 \times 10^5$$

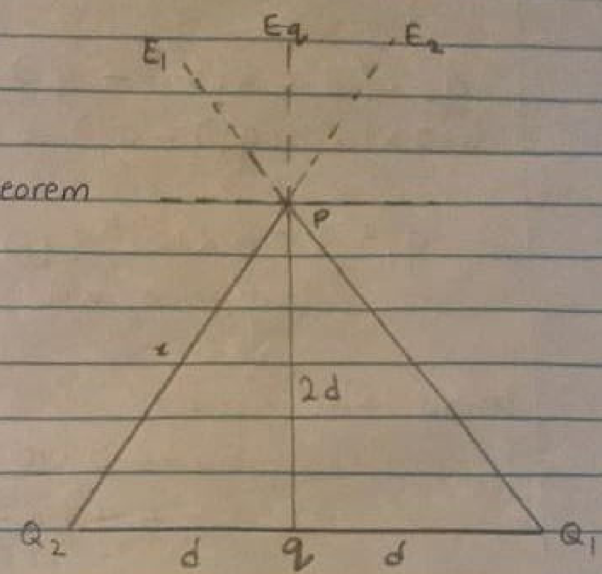
$$q = \frac{-1.026 \times 10^5}{9.0 \times 10^9}$$

$$q = -11.4 \mu C$$

If $q_1 = 3.84 \times 10^{-5}$ C, $q_2 = 5.0 \times 10^{-5} - 3.84 \times 10^{-5}$ C
$q_2 = 1.16 \times 10^{-5}$ C

If $q_1 = 1.16 \times 10^{-5}$ C, $q_2 = 5.0 \times 10^{-5} - 1.16 \times 10^{-5}$ C
$q_2 = 3.84 \times 10^{-5}$ C.

1c. $q_1 = q_2 = 8\,\mu C$
$d = 0.5m$



To find x, use pythagoras theorem

$x^2 = d^2 + 2d^2$

$x^2 = 0.5^2 + (2(0.5))^2$

$x^2 = 0.25 + 1$

$x^2 = 1.25$

$x = \sqrt{1.25} = 1.12m$

$\sin\theta = \frac{1}{1.12}, \theta = 63.43°$

$\cos\theta = \frac{0.5}{1.12} \Rightarrow \theta = 63.43°$ (for cross checking)

$$E_1 = \frac{kq}{r_1^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5.74 \times 10^4\ N.C^{-1}$$

$$E_2 = \frac{kq}{r_2^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5.74 \times 10^4\ N.C^{-1}$$

$$E_q = \frac{kq}{r^2} = \frac{9 \times 10^9 \times q}{1} = 9 \times 10^9 . q$$

vector	θ	x-comp	y-comp
$E_1 = 5.74 \times 10^4\ N.C^{-1}$	63.43°	$5.74 \times 10^4 \cos 63.43 = 2.57 \times 10^4$	$5.74 \times 10^4 \sin 63.43 = 5.13 \times 10^4$
$E_2 = 5.74 \times 10^4\ N.C^{-1}$	63.43°	$5.74 \times 10^4 \cos 63.43 = -2.57 \times 10^4$	$5.74 \times 10^4 \sin 63.43 = 5.13 \times 10^4$
$E_q = 9 \times 10^9 . q\ N.C^{-1}$	90°	$9 \times 10^9 . q \cos 90 = 0$	$9 \times 10^9 . q \sin 90 = 9 \times 10^9 . q$
		$\Sigma x = 0\ N.C^{-1}$	$E_y = 1.026 \times 10^5\ N.C^{-1} + 9.0 \times 10^9 q\ N.C^{-1}$

$$q_1 \cdot q_2 = \frac{(1.0)(2.0)^2}{8.99 \times 10^9}$$

$$q_1 \cdot q_2 = 4.45 \times 10^{-10}\ C^2$$

Since $q_1 + q_2 = 5.0 \times 10^{-5}\ C$

$$\longrightarrow q_2 = 5.0 \times 10^{-5}\ C - q_1$$

$$q_1 \cdot (5.0 \times 10^{-5} - q_1) = 4.45 \times 10^{-10}$$

$$5.0 \times 10^{-5}(q_1) - q_1^2 = 4.45 \times 10^{-10}$$

$$q_1^2 - 5.0 \times 10^{-5}(q_1) + 4.45 \times 10^{-10} = 0$$

Compare with $ax^2 + bx + c = 0$

Using that, $a = 1$

$b = -5.0 \times 10^{-5}$

$c = 4.45 \times 10^{-10}$

Using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q_1 = \frac{-(-5.0 \times 10^{-5}) \pm \sqrt{(-5.0 \times 10^{-5})^2 - 4(1)(4.45 \times 10^{-10})}}{2(1)}$$

$$q_1 = \frac{5.0 \times 10^{-5} \pm \sqrt{25 \times 10^{-10} - 17.8 \times 10^{-10}}}{2}$$

$$q_1 = \frac{5.0 \times 10^{-5} \pm \sqrt{7.2 \times 10^{-10}}}{2}$$

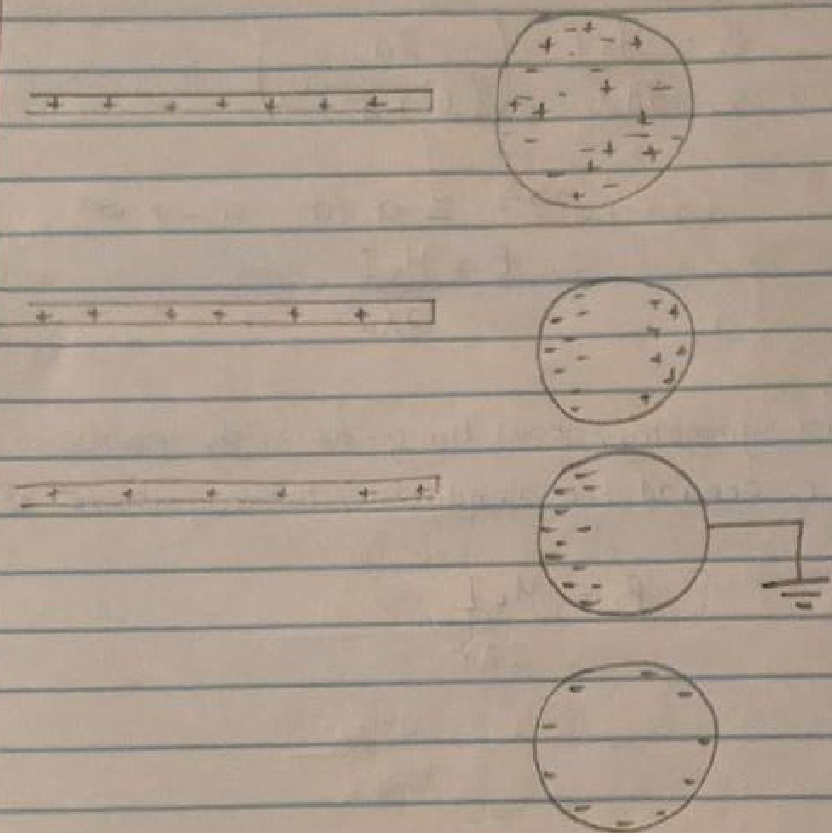
$$q_1 = \frac{5.0 \times 10^{-5} \pm 2.68 \times 10^{-5}}{2}$$

$$q_1 = \frac{5.0 \times 10^{-5} + 2.68 \times 10^{-5}}{2} \quad \text{OR} \quad q_1 = \frac{5.0 \times 10^{-5} - 2.68 \times 10^{-5}}{2}$$

$$q_1 = \frac{7.68 \times 10^{-5}}{2} \qquad q_1 = \frac{2.32 \times 10^{-5}}{2}$$

$$q_1 = 3.84 \times 10^{-5}\ C \qquad q_1 = 1.16 \times 10^{-5}$$

the earth. If the wire to ground is then removed, the conducting sphere is left with an excess of induced negative charge. When the rod is removed from the vicinity of the sphere, the induced negative charge remains on the ungrounded sphere, uniformly distributed over the surface of the sphere.



b. $q_1 + q_2 = 5.0 \times 10^{-5}\ C$

$F = 1.0\ N$

$r = 2.0\ m$

$$F = K \frac{q_1 q_2}{r^2}$$

$$q_1 \cdot q_2 = \frac{F \cdot r^2}{K}$$

3 becomes.

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2 (x^2+y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2 (x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

$$(x^2+a^2)^{1/2} \cong a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

Due to axial symmetry, about the y-axis, all points in a circle of radius r, around the conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r}$$

1. Charging by induction

Consider a positively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conducting path to ground. The repulsive force between the electrons in the rod and those in the sphere cause a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod. The region of the sphere nearest the positively charged rod has an excess of negative charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, some of the electrons leave the sphere, travelling to

Applying Biot-Savart Law, the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From the diagram, $r^2 = x^2 + y^2$ (using pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad 1$$

But $\sin(\pi - \varphi) = \dfrac{x}{\sqrt{x^2+y^2}} = \dfrac{x}{(x^2+y^2)^{1/2}}$ 2

Substitute ② into ①,

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl}{(x^2+y^2)} \cdot \frac{x}{(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy$$

Using special integrals,

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}} \quad 3$$