The Superposition Principle

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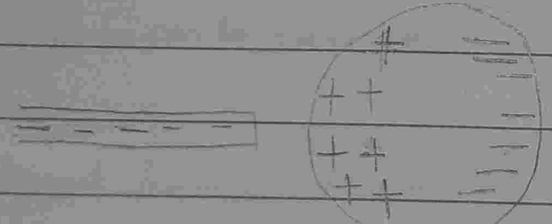
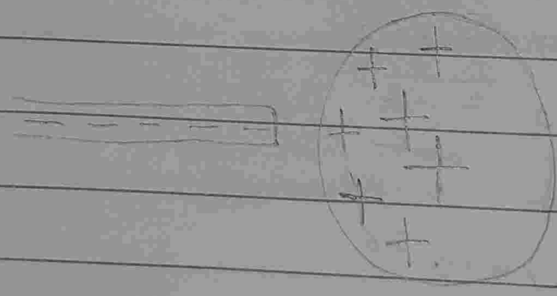
COURSE: PHY 102

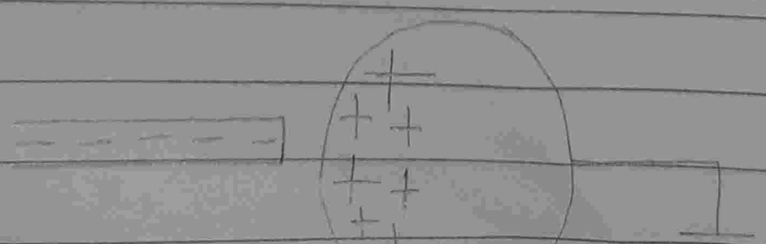
Assignment

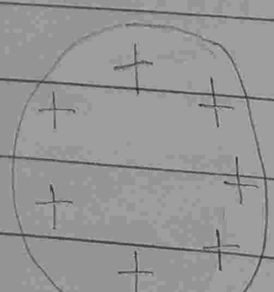
1(a) How to produce a negatively charged sphere by induction

A negatively charged rubber rod is brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod. The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed, the conducting sphere is left with excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere, the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.









b.) Each of two small spheres is charged positively, the combined charge being $5.0 \times 10^{-5}C$. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m ~~apart~~ apart, calculate the charge on each sphere.

Solution

$q_1 + q_2 = 5.0 \times 10^{-5}C$

$f = 1.0N$; $K = 8.99 \times 10^9 Nm^2/C^2$

$r = 2.0m$

$F = \frac{Kq_1q_2}{r^2}$

$1.0 = \frac{8.99 \times 10^9 \times q_1q_2}{2^2}$

$1.0 = \frac{8.99 \times 10^9 \times q_1q_2}{4}$

$4.0 = 8.99 \times 10^9 \times q_1q_2$

$q_1q_2 = \frac{4.0}{8.99 \times 10^9}$

$q_1q_2 = 4.449 \times 10^{-10}C^2$ --- (1)

$q_1 + q_2 = 5.0 \times 10^{-5}C$ ------(11)

$q_1 = 5.0 \times 10^{-5} - q_2$

Put $q_1 = 5.0 \times 10^{-5} - q_2$ into equation (1)

$q_1q_2 = 4.449 \times 10^{-2}C^2$

$(5.0 \times 10^{-5} - q_2)q_2 = 4.449 \times 10^{-10}$

$5.0 \times 10^{-5}q_2 - q_2^2 = 4.449 \times 10^{-10}$

$5.0 \times 10^{-5}q_2 - q_2^2 - 4.449 \times 10^{-10} = 0$

$-q_2^2 + 5.0 \times 10^{-5}q_2 - 4.449 \times 10^{-10} = 0$

$q_2^2 - 5.0 \times 10^{-5}q_2 + 4.449 \times 10^{-10} = 0$

Using quadratic formula

$a = 1$, $b = 5.0 \times 10^{-5}$, $c = 4.449 \times 10^{-10}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5 \times 10^{-5}) \pm \sqrt{(-5.0 \times 10^{-5})^2 - 4(1 \times 4.449 \times 10^{-10}}}{2(1)}$

$$x = \frac{5\times10^{-5} \pm \sqrt{2.5\times10^{-9} - 4(4.449\times10^{-10})}}{2}$$

$$x = \frac{5\times10^{-5} \pm \sqrt{2.5\times10^{-9} - 1.7796\times10^{-9}}}{2}$$

$$x = \frac{5\times10^{-5} \pm \sqrt{7.204}}{2}$$

$$x = \frac{5\times10^{-5} \pm 2.68\times10^{-5}}{2}$$

$$x = \frac{5\times10^{-5} + 2.68\times10^{-5}}{2} \text{ or } \frac{5\times10^{-5} - 2.68\times10^{-5}}{2}$$

$x = 3.84\times10^{-5}$ or 1.16×10^{-5}

$q_2 = 3.84\times10^{-5}$ or 1.16×10^{-5}

When $q_2 = 3.84\times10^{-5}$

$q_1 = 5.0\times10^{-5} - 3.84\times10^{-5}$

$q_1 = 1.16\times10^{-5}$

When $q_2 = 1.16\times10^{-5}$

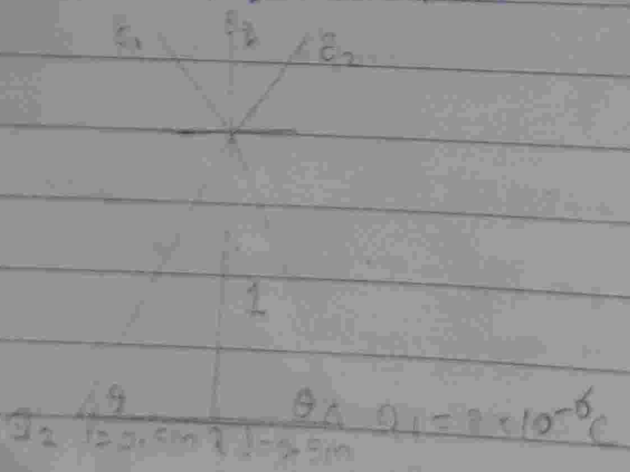
$q_1 = 5.0\times10^{-5} - 1.16\times10^{-5}$

$q_1 = 3.84\times10^{-5}$

When $q_1 = 3.84\times10^{-5}$, $q_2 = 1.16\times10^{-5}$

$q_1 = 1.16\times10^{-5}$, $q_2 = 3.84\times10^{-5}$

(c)



$Q_1 = Q_2 = 8\,\mu C$

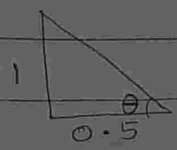
$d = 0.5\,m$

$$\tan\theta = \frac{1}{0.5}$$

$\tan\theta = 2$

$\theta = \tan^{-1} 2$

$\theta = 63.4349$



$H^2 = 1^2 + 0.5^2$

$H^2 = 1 + 0.25$

$H = \sqrt{1.25}$

$H = 1.118 = r$

$E_1 = \frac{kQ_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.118)^2} = \frac{7200}{1.249924} = 5760.350229\,C$

$E_2 = \frac{kQ_2}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.118)^2} = \frac{7200}{1.249924} = 5760.350229\,C$

$E_q = \frac{kq}{r^2} = \frac{9\times10^9 q}{1^2} = 9\times10^9 q\,C$

Vector	Angle	x-component	y-component
$E_1 = 5760.350229$	63.4349^0	$E_1 \cos\theta = 111.328$	$E_1 \sin\theta = 5152.211679$
$E_2 = 5760.350229$	63.4349^0	$E_2 \cos\theta = 111.328$	$E_2 \sin\theta = 5152.211679$
$E_q = 9\times10^9 q$	90^0	$E_q \cos\theta = 0$	$E_q \sin\theta = 9\times10^9 q$
		$\Sigma x = 0$	$E_y = 10304.42336$

Magnitude $= \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

$E_q = \sqrt{0^2 + (10304.42336)^2}$

Since $E_q = 0$

$0 = 9\times10^9 q + 10304.42336$

$q = \frac{-10304.42336}{9\times10^9}$

$q = -1.1449\times10^{-6}$

$q = -1.1\,\mu C$

2a) Electric field

An electric field is a region of space in which an electric charge will experience an electric force.

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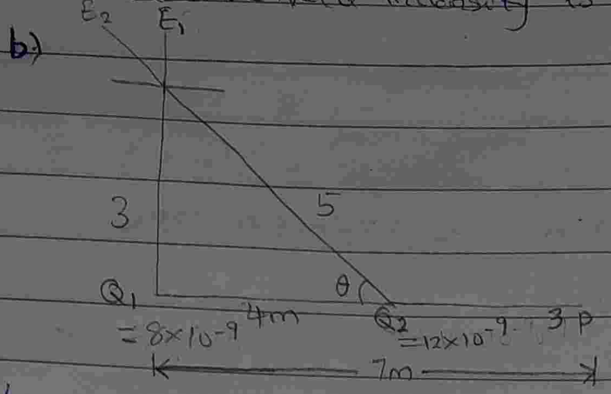
Electric field intensity

The electric field intensity is defined as the force per unit charge.

b)



$\tan\theta = \frac{3}{4}$ $Q_1 = 8\times10^{-9}$

$Q_2 = 12\times10^{-9}$

$\tan\theta = 0.75$

$\theta = \tan^{-1} 0.75$

$\theta = 36.8699$

~~$E_{net} = E_1 + E_2$~~

$E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^9\times8\times10^{-9}}{7^2} = \frac{72}{49} = 1.4694\ N/C$

$E_2 = \frac{KQ_2}{r^2} = \frac{9\times10^9\times12\times10^{-9}}{3^2} = \frac{108}{9} = 12\ N/C$

(i) $E_{net} = E_1 + E_2$

$E_{net} = 1.4694 + 12$

$E_{net} = 13.4694\ N/C$

$E_{net} = 13.5\ N/C$

ii) $E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^9\times8\times10^{-9}}{3^2} = \frac{72}{9} = 8\ N/C$

$E_2 = \frac{KQ^2}{r^2} = \frac{9\times10^9\times12\times10^{-9}}{5^2} = \frac{108}{25} = 4.32\ N/C$

Vector	Angle	x-component	y-component
$E_1 = 8\ N/C$	$90°$	$E_1 \cos\theta = 0$	$E_1 \sin\theta = 8$
$E_2 = 4.32\ N/C$	$36.8699°$	$E_2 \cos\theta = -3.455$	$E_2 \sin\theta = 2.592$
		$\Sigma E_x = -3.455$	$\Sigma E_y = 10.592$

$E_{net} = \sqrt{\Sigma E_x^2 + \Sigma E_y^2}$

$E_{net} = \sqrt{(-3.455)^2 + (10.592)^2}$

$E_{net} = \sqrt{11.937025 + 112.190464}$

$E_{net} = \sqrt{124.127489}$

$E_{net} = 11.14125$

$E_{net} = 11.14\ N/C$

4a) Magnetic flux is defined as the strength of magnetic field ~~represented~~ by lines of force. It is usually represented by the symbol ϕ.

b) $m_e = 9.11 \times 10^{-31}\ kg$

$r = 1.4 \times 10^{-7}\ m$

$B = 3.5 \times 10^{-1}\ weber/meter^2$

$\omega = ?$

$\omega = \frac{v}{r} = \frac{qB}{m_e} = \frac{-1.60 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$

$\omega = -6.147 \times 10^{11}\ rad/s$

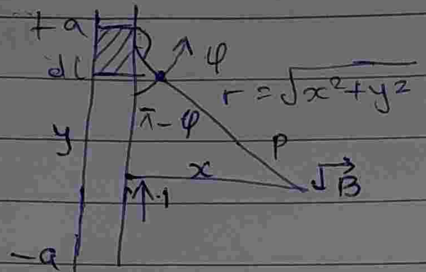
c) The answer is negative since it is an electron.

5a) Biot Savart Law

The Biot-Savart law is an equation that describes the magnetic field ~~...~~ created by a current carrying wire and allows you to calculate its strength at various points

$$B = \frac{\mu_0}{4\pi} \int \frac{I\,dl \sin\theta}{r^2}$$

b)



Applying the Biot-Savart law, we find the magnitude of the field $\vec{dB}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x + y^2} \quad \text{--- (1)}$$

But $\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$ --- (2)

Substituting equation (2) into (1), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad --- (3)$$

Using special integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation (3) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length 2a of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is when a is much larger than x,

$$(x^2+a^2)^{1/2} \simeq a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y-axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is:

$$B = \frac{\mu_0 I}{2\pi r}.$$