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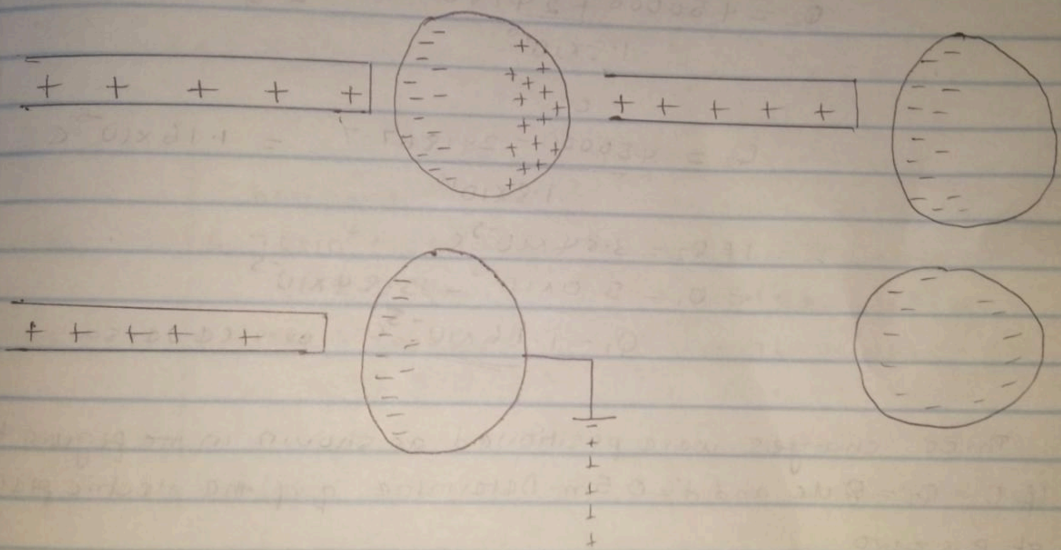
19/MHS01/198
MBBS
Phy 102

Assignment

1a Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

Ans:



1b Each of two small spheres is charged positively, the combined charge being $5.0 \times 10^{-5}C$. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart calculate the charge on each sphere

Ans: $F = 1N$, $r = 2m$, $Q_1 = ?$, $Q_2 = ?$

$$F = \frac{Q_1 Q_2 K}{r^2}$$

But $Q_1 + Q_2 = 5.0 \times 10^{-5}C$

$$Q_1 = 5.0 \times 10^{-5}C - Q_2$$

$$\therefore 1 = \frac{(5.0 \times 10^{-5}C - Q_2)Q_2 \times 9 \times 10^9}{2^2}$$

$$4 = (5.0 \times 10^{-5}C - Q_2)\, 9 \times 10^9 Q_2$$

$$4 = 450000Q_2 - 9\times10^9 Q_2^2$$

$$9\times10^9 Q_2^2 - 450000Q_2 + 4 = 0$$

$$Q_2 = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$Q_2 = \frac{-(-450000) \pm \sqrt{(450000)^2 - 4(9\times10^9)\times4}}{2(9\times10^9)}$$

$$Q_2 = \frac{450000 \pm \sqrt{5.85\times10^{10}}}{1.8\times10^{10}}$$

$$Q_2 = \frac{450000 + 241867.7}{1.8\times10^{10}} = 3.84\times10^{-5}\,C$$

or

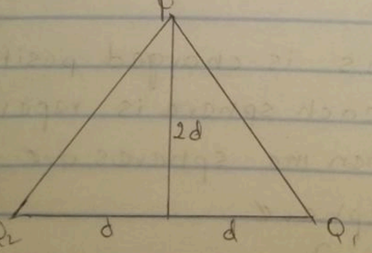
$$Q_2 = \frac{450000 - 241867.7}{1.8\times10^{10}} = 1.16\times10^{-5}\,C$$

$\therefore$ If $Q_2 = 3.84\times10^{-5}\,C$.

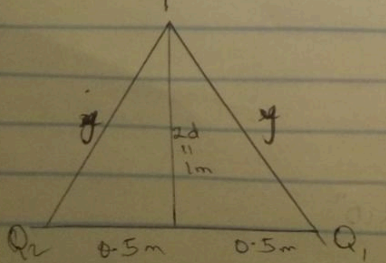
$$Q_1 = 5.0\times10^{-5} - 3.84\times10^{-5}$$

$$Q_1 = 1.16\times10^{-5}\,C$$ or vice versa

1c Three charges were positioned as shown in the figure below. If $Q_1 = Q_2 = 8\,\mu C$ and $d = 0.5m$. Determine q if the electric field at P is zero



Ans:



$x = \tan^{-1}(1/0.5)$

$x = 63.435^\circ \simeq 63.44^\circ$

$y^2 = \sqrt{0.5^2 + 1^2}$

$y = 1.118m$

components,

$E_1 = \frac{KQ}{r^2} = \frac{1Q}{4\pi\varepsilon_0 r^2} = \frac{8\times10^{-6}\times9\times10^9}{1.118^2} = 5.76\times10^4$

$E_2 = \frac{KQ}{r^2} = \frac{8\times10^{-6}\times9\times10^9}{1.118} = 5.76\times10^4$

$E_q = \frac{x(9\times10^9)}{1^2} = 9\times10^9 x$

	horizontal	vertical
E_1	$5.76\times10^4\cos63.4\cancel{35}$ $= 2.574\times10^4$	$5.76\times10^4\sin63.44$ 5.152×10^4
E_2	$-5.76\times10^4\cos63.44$ $= -2.574\times10^4$	$5.76\times10^4\sin63.44$ $= 5.152\times10^4$
E_q	0	$\sin90\times9\times10^9 x = 9\times10^9 x$
	$E_{fx} = 0$	$E_{fy} = 10.3040 + 9\times10^9 x$

$E = \sqrt{(E_{fx})^2 + (E_{fy})^2}$

$E = \sqrt{(0)^2 + (10.3040 + 9\times10^9 x)^2}$

from the question, $E = 0$ at point P

$0 = \sqrt{0 + (10.3040 + 9\times10^9 x)^2}$

$0 = 10.3040 + 9\times10^9 x$

$\frac{-9\times10^9 x}{-9\times10^9} = \frac{10.3040}{-9\times10^9}$

$x = -1.1\times10^{-5}C = -11\mu C$

3 ~~Distinguish~~ State the formulation of the following identities of charges

i) volume charge density: $\rho = \frac{dQ}{dv} \Rightarrow dQ = \rho dv$

ii) Surface charge density: $\sigma = dQ/da \Rightarrow dQ = \sigma da$

3b Explain with appropriate equations, the electric potential difference between two points in an electric field

Ans: The electric potential difference between two points in an electric field is the work done per unit charge against the electric forces when a charge is transported from one point to another. Its measurement is in volts (V) or joules per columb (J/C). It is a scalar quantity. To move a test charge from one point to another at constant velocity, an external force of $F = -q_0 E$ must act on the charge. Therefore, the work done, dW, is given as

$$dW = F \cdot dl \quad \text{—(i)}$$

but $F = -q_0 E$ —(ii)

substituting eqn (ii) in (i)

$$dW = -q_0 E dl \quad \text{—(iii)}$$

The total work done in moving the test charge from A - B is

$$W(A \rightarrow B)_{Ag} = -q_0 \int_A^B E dl \quad \text{—(iv)}$$

From the definition

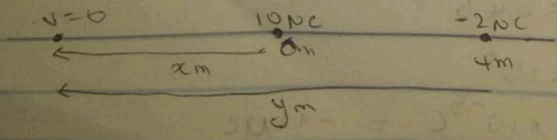
$$V_B - V_A = \frac{W(A \rightarrow B)_{Ag}}{q_0} \quad \text{—(v)}$$

putting (iv) in (v)

$$V_B - V_A = -\int_A^B E dl \quad \text{—(vi)}$$

3c Two point charges $Q = 10NC$ and $Q_2 = -2NC$ are arranged along the x-axis at $x = 0$ & $x = 4m$ respectively. Find the position along the x-axis where $V = 0$

Soln



$y_m = x_m + 4$

$$V = k\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right)$$

$Q = 10NC, r_1 = x$, $Q_2 = -2NC, r_2 = x + 4$

$\therefore 0 = 9\times10^9\left(\frac{1.0\times10^{-5}}{x} + \frac{(-2\times10^{-6})}{x+4}\right)$ divide both sides by 9×10^9

$0 = \frac{1.0\times10^{-5}}{x} - \frac{2\times10^{-6}}{x+4}$

$\frac{2\times10^{-6}}{x+4} = \frac{1\times10^{-5}}{x}$

$2\times10^{-6}x = 1\times10^{-5}x + 4\times10^{-5}$ collect like terms

$2\times10^{-6}x - 1\times10^{-5}x = 4\times10^{-5}$

$\frac{-8\times10^{-6}x}{-8\times10^{-6}} = \frac{4\times10^{-6}}{-8\times10^{-6}}$

$x = -5$

$x = -5, y = -5+4 \quad \therefore y = -1$

The distances are -5 and 1

SECTION B

4a What is magnetic flux?

Ans Magnetic flux through a surface is the surface integral of the normal component of the magnetic field flux density, B, passing through the surface.

4b An electron with a vast mass of 9.11×10^{-31} kg moves in a circular orbit of radius 1.4×10^{-7} m in a uniform magnetic field of 3.5×10^{-1} weber/meter square, perpendicular to the speed with which electron moves. Find the cyclotron frequency of the moving electron.

Soln $q = 1.60\times10^{-19}$ C, $r = 1.4\times10^{-7}$ m, $B = 3.5\times10^{-1}$, $m_e = 9\times11\times10^{-31}$ kg

$v = \frac{qBr}{m_e} = \frac{1.6\times10^{-19}(3.5\times10^{-1})\times1.4\times10^{-7}}{9.11\times10^{-31}}$

$v = 8.6\times10^{3}$ m/s

$\omega = \frac{v}{r} = \frac{8.6\times10^{3}}{1.4\times10^{-7}} = \underline{6.14\times10^{10}\text{ rad/s}}$

4c Discuss the answer in 4B above

Ans The answer above implies that at the frequency of the electron to move perpendicular to the distance of the uniform magnetic field B is 6.14×10^{10} rad/s.

5 State the Biot-Savart law

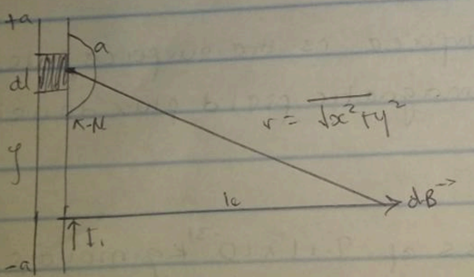
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

where μ_0 is a constant called permeability of free space

$$\mu_0 = 4\pi \times 10^{-7}\ T.m/A$$

5b Using the Biot-Savart law, show the magnitude of the magnetic field of a straight current carrying conductor given as $B = \frac{\mu_0 I}{2\pi r}$

Soln:



Applying the Biot-Savart law,

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \theta) = \sin\theta$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{+a} \frac{dl \sin(\pi - \theta)}{r^2}$$

from the above diagram $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{2\pi} \int_{-a}^{+a} \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \quad \text{—(i)}$$

But $\sin(\pi - \theta) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$ —(ii) substitute (ii) in (i)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{+a} dl \, \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

remember, $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{+a} \frac{x}{(x^2+y^2)^{3/2}}$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{+a} \frac{1}{(x^2+y^2)^{3/2}}\,dy \longrightarrow \text{(iii)}$$

using special integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\,\frac{y}{(x^2+y^2)^{1/2}}$$

∴ eqn (iii) becomes

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{y}{x^2(x^2+y^2)^{1/2}}\right)_{-a}^{+a} = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length 2a of a conductor is very great in comparison to its distance x from point P, we consider it infinitely long (i.e its much longer than x).

$$(x^2+a^2)^{1/2} \cong a, \; a \leq a \longrightarrow x$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, there is axial symmetry about the y-axis. Therefore, at all points in a circle of radius r, around the conductor of magnitude B is

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$