①

a. Electric charges can be obtained on an object without touching it using a process known as electrostatic induction. Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to the ground. The repulsive force between the electrons in the rod and those in the sphere will cause a redistribution of charges so that negative electrons move to the side of the sphere farthest away from the rod and the region of the sphere nearest the negatively charged rod has an excess of positive charges. A grounded wire is then connected to the sphere and the negative electrons in the sphere leave the sphere and travel to the earth. The wire to the ground in then removed and the sphere is left with excess positive charge. And when the rubber rod is finally removed, the induced positive charge remains on the sphere and becomes evenly distributed.

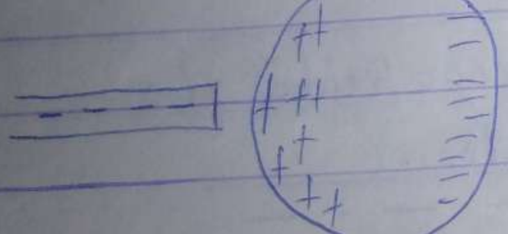


Figure (a)

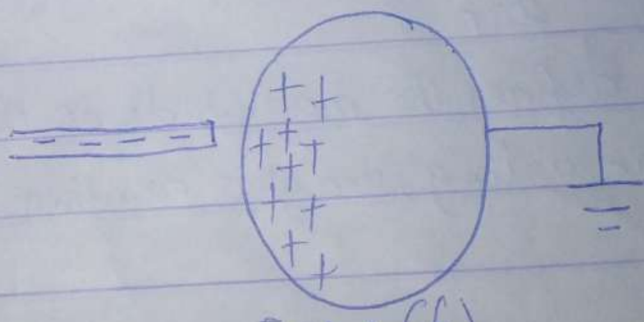


Figure (b)

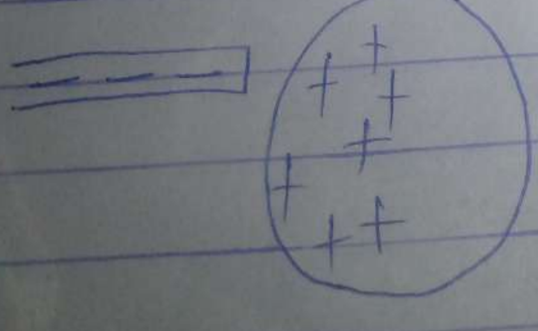


Figure (c)

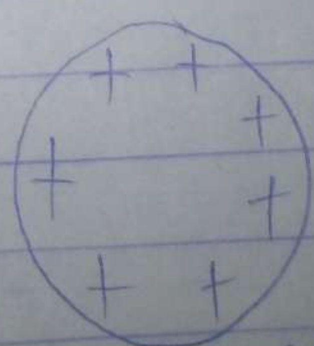


Figure (d)

b. $F = 1N$

$d = 2m$

$q_1 + q_2 = 5\times10^{-5}\,C$

$q_1 = 5\times10^{-5} - q_2$

$k = 9\times10^9$

$$F = \frac{kq_1q_2}{d^2}$$

$$1 = \frac{9\times10^9\times(5\times10^{-5} - q_2)\times q_2}{(2)^2}$$

$$4 = (9\times10^9\times5\times10^{-5}\times q_2) - (9\times10^9\times q_2\times q_2)$$

$$4 = 4.5\times10^5 q_2 - 9\times10^9 q_2^2$$

$$9\times10^9 q_2^2 - 4.5\times10^5 q_2 + 4 = 0$$

Solving using the quadratic formula,

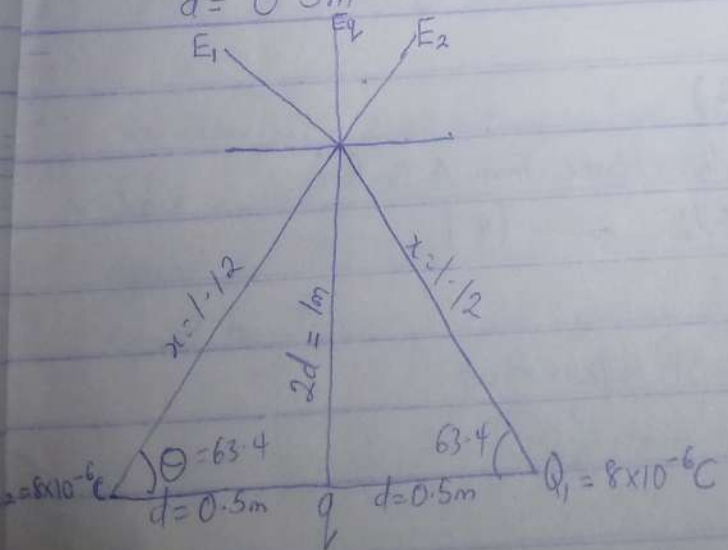
$q_1 = 3.8\times10^{-5}\,C$

$q_2 = 1.2\times10^{-5}\,C$.

Hence, the charge on each sphere is $3.8\times10^{-5}\,C$ and $1.2\times10^{-5}\,C$ respectively.

c. $Q_1 = Q_2 = 8\mu C$

$d = 0.5m$



$\tan\theta = \frac{1}{0.5}$

$\theta = \tan^{-1}(2)$

$\theta = 63.4$

$x^2 = 1^2 + 0.5^2$

$x = \sqrt{1.25}$

$x = 1.12$

$$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9\times8\times10^{-6}}{(1.12)^2} = 5.7\times10^4$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9\times8\times10^{-6}}{(1.12)^2} = 5.7\times10^4$$

$$E_q = \frac{kq}{r^2} = \frac{9\times10^9\times q}{1} = 9\times10^9 q$$

Vector	Angle	X-component	Y-component
$E_1 = 5.7\times10^4$	63.4	$-5.7\times10^4\cos63.4$ $= -2.6\times10^4$	$5.7\times10^4\sin63.4$ $= 5.1\times10^4$
$E_2 = 5.7\times10^4$	63.4	2.6×10^4	5.1×10^4
$E_q = 9\times10^9 q$	90°	0	$9\times10^9 q$
		$\Sigma x = 0$	$\Sigma y = 102000$

$$\text{Magnitude} = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$E_q = \sqrt{(0)^2 + (102000)^2}$$

since $E = 0$

$0 = 9\times10^9 q + 102000$

$9\times10^9 q = -102000$

$$q = \frac{-102000}{9\times10^9}$$

$q \simeq 11.4\mu C$.

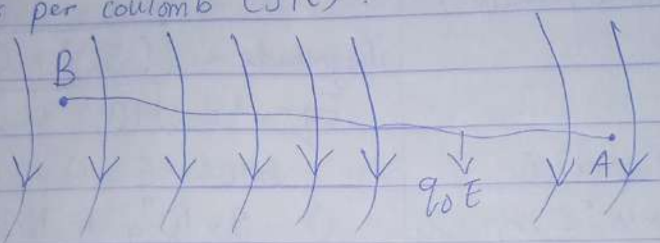
a. (i) Volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

(ii) Surface charge density, $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

(iii) Linear charge density, $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

b. ELECTRIC POTENTIAL DIFFERENCE

The electric potential difference between two points in an electric field can be defined as the workdone per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (v) or Joules per coulomb (J/c).



In the diagram above, to move the test charge from A to B at constant velocity, an external force of $F = -q_0E$ must act on the charge. Therefore the elemental work done dW is given as: $dW = F \cdot dL$ — (1)

But $F = -q_0E$ — (2)

Substituting (2) into (1) yields

$$dW = -q_0 E dL \quad \text{— (3)}$$

Then the total work done in moving the test charge from A to B is.

$$W(A \rightarrow B)_{Ag} = -q_0 \int_A^B E dL \quad \text{— (4)}$$

From the definition of potential difference, it follows that:

$$V_B - V_A = \frac{W(A \rightarrow B)_{Ag}}{q_0} \quad \text{— (5)}$$

Putting (4) in (5) yields $V_B - V_A = -\int_A^B E dL$ — (6)

a. Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol Φ. Mathematically given as $\Phi = B \cdot dA$.

b. $M = 9 \times 10^{-31}\,kg$

$r = 1.4 \times 10^{-7}\,m$

$B = 3.5 \times 10^{-1}$ weber/meter^2

cyclotron frequency = angular speed

$$W = \frac{v}{r} = \frac{qB}{m}$$

$$W = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$W = 62222222222.22222\,T^{-1}$$

c. In the question, the parameters given were

i. Radius = $1.4 \times 10^{-7}\,m$

ii. mass of electron = $9.11 \times 10^{-31}\,kg$

iii. Magnetic field of 3.5×10^{-1} weber/meter^2.

And the question being asked was to find the cyclotron frequency which is equal to angular speed.

Recall that angular speed w is given as $w = \frac{v}{r}$

So therefore, substituting values gives the final answer to be 62222222222.222221^{-1}.

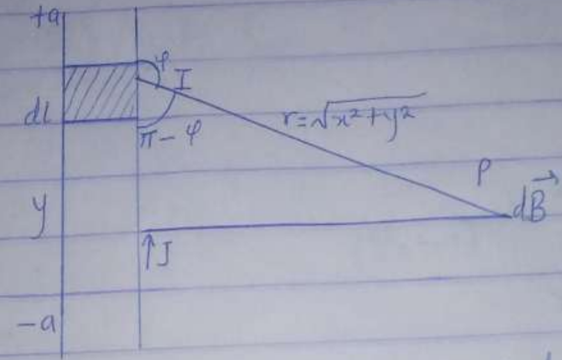
Since cyclotron frequency is equal to angular speed, the cyclotron frequency is equal to 62222222222.222221^{-1} having a unit as 1/T which is equal to the unit of frequency dimensionally.

(a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ), the current (I), the change in length, the radius and inversely proportional to the square of radius (r^2). It can be represented mathematically by

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^2}$$

where μ_0 is a constant called Permeability of free space.

(b) MAGNETIC FIELD OF A STRAIGHT CURRENT CARRYING CONDUCTOR



Applying the Biot-Sarvart law, we find the magnitude of the field $\vec{dB}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{——— (1)}$$

$$\text{But } \sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{——— (2)}$$

Substituting (2) into (1), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dL = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \text{———} \quad (3)$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation (3) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length 2a of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is, when a is much larger than x,

$$(x^2+a^2)^{1/2} \cong a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y-axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is $B = \frac{\mu_0 I}{2\pi r}$ ——— (4)

Equation (4) defines the magnitude of the magnetic field of flux density B near a long, straight carrying conductor.