DARE BENEDICT OLUBUKOLA
MECHANICAL ENGINEERING
19/ENG06/016
PHY 102 (COVID-19 HOLIDAY ASSIGNMENT)

SECTION A

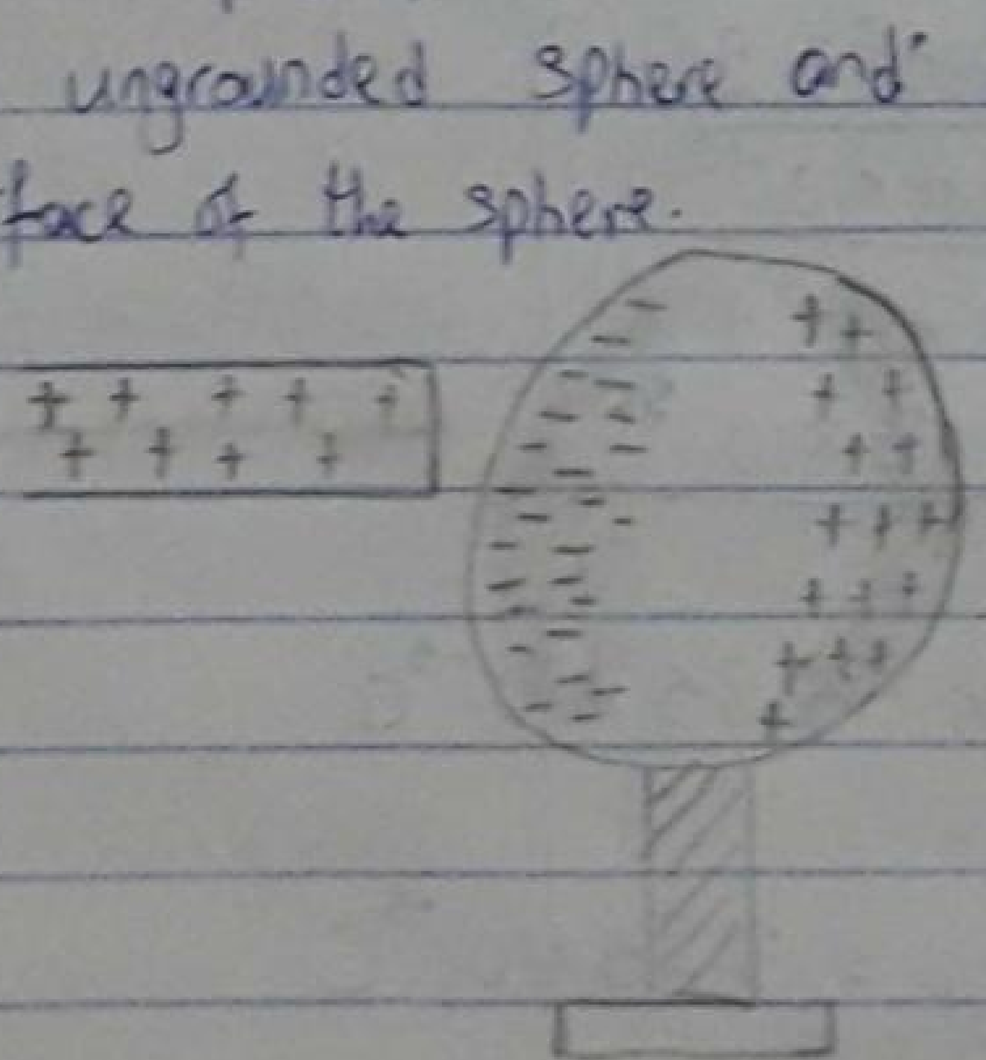
1a. Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction

Answer

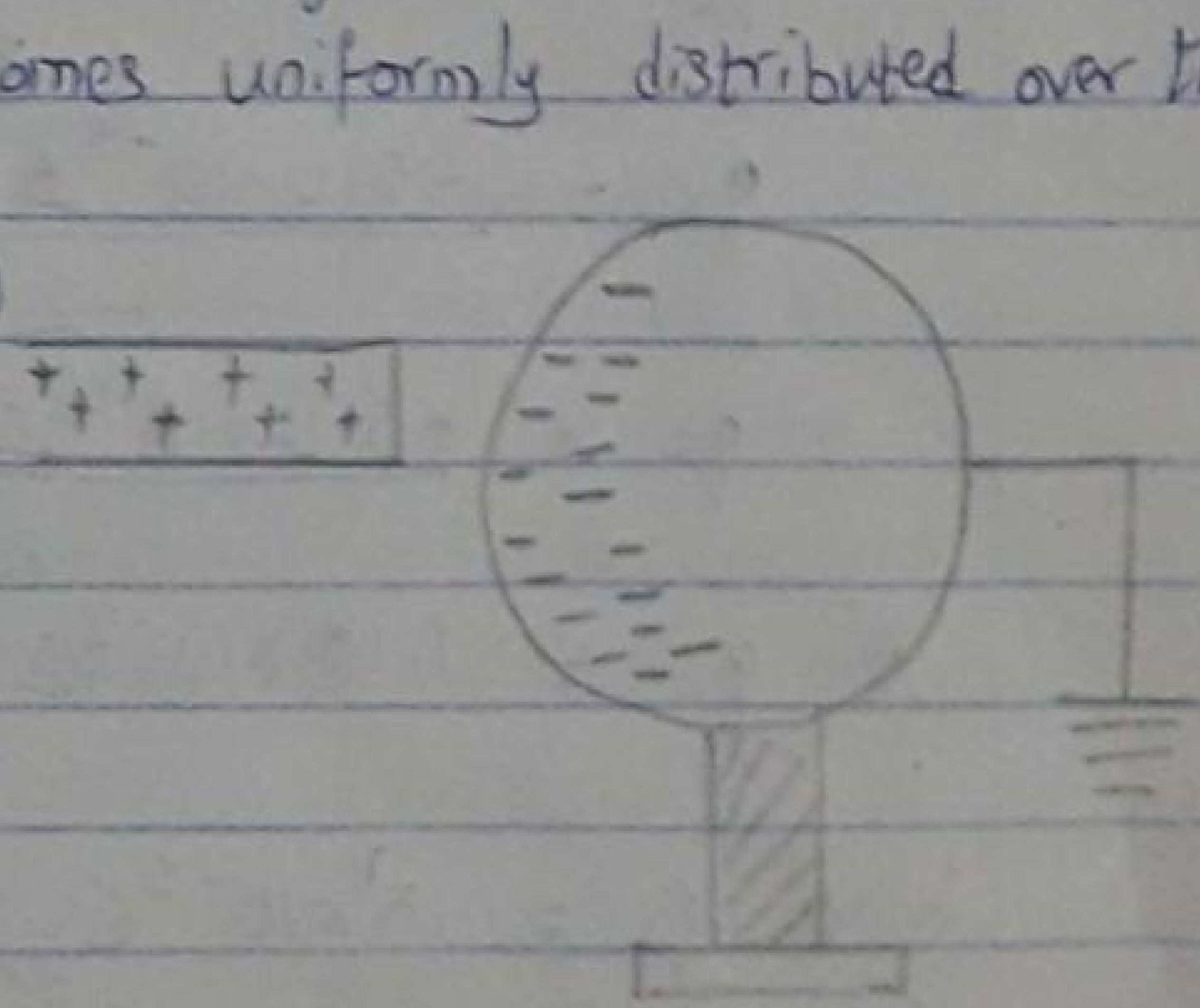
Consider a positively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conducting path to ground. The repulsive force between the ~~electrons~~ protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to the positively charged rod has an excess of electrons because of the migration of protons away from this location. If a grounded conducting wire is then connected to the sphere, some of the protons leave the sphere and travel to the earth. With the wire to the ground removed, the conducting sphere is left with an excess of induced negative charge.

Finally, when the rubber rod is removed from the vicinity of the sphere, the induced negative charge (electrons) remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

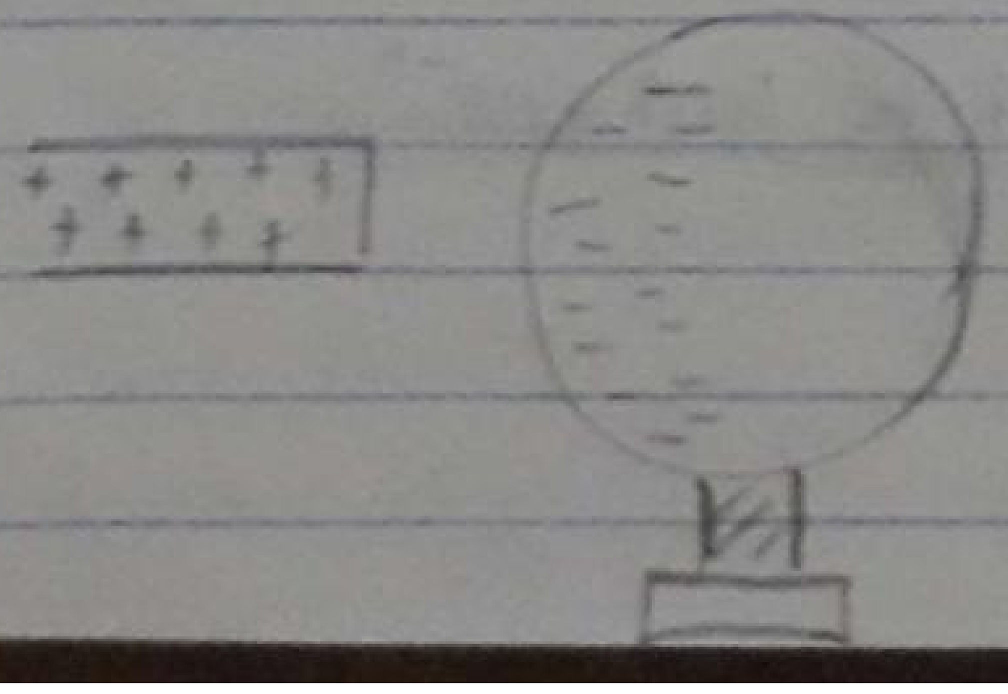
(a)



(b)



(c)



(d)

16. Each of two small spheres is charged positively, the combined charge being $5.0 \times 10^{-5} C$. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2m apart, calculate the charge on each sphere.

<u>Solution</u>

$$q_1 + q_2 = 5.0 \times 10^{-5} C$$

$$F = 1.0N$$

$$r = 2m$$

$$F = \frac{kq_1q_2}{r^2}$$

$$\Rightarrow \quad 1 = \frac{9 \times 10^9 \times q_1 q_2}{2^2}$$

$$q_1 q_2 = \frac{4}{9 \times 10^9}$$

$$q_1 q_2 = 4.4 \times 10^{-10}$$

But,

$$q_1 = 5.0 \times 10^{-5} - q_2$$

$$\therefore \quad (5.0 \times 10^{-5} - q_2) q_2 = 4.4 \times 10^{-10}$$

$$5.0 \times 10^{-5} q - q_2^2 = 4.4 \times 10^{-10}$$

$$-q_2^2 + 5.0 \times 10^{-5} q - 4.4 \times 10^{-10} = 0$$

Using quadratic formula;

$$q_2 = \frac{-5.0 \times 10^{-5} \pm \sqrt{(5.0 \times 10^{-5})^2 - 4[(-1)(-4.4 \times 10^{-10})]}}{2(-1)}$$

$$q_2 = \frac{-5.0 \times 10^{-5} \pm \sqrt{7.4 \times 10^{-10}}}{-2}$$

$$q_2 = \frac{-5.0 \times 10^{-5} \pm 2.7 \times 10^{-5}}{-2}$$

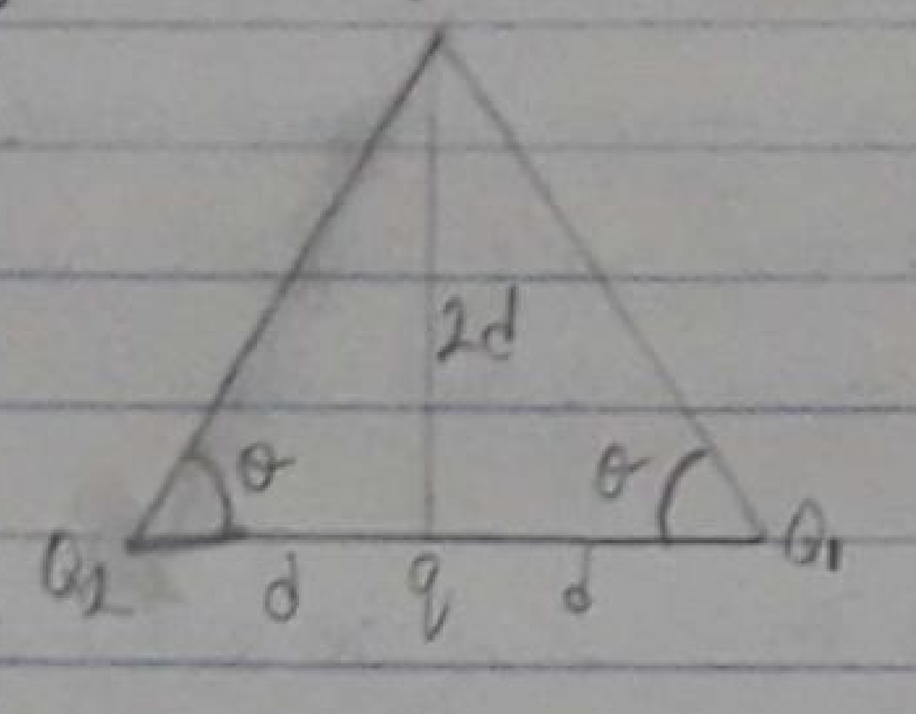
$$\therefore \quad q_2 = 1.15 \times 10^{-5} C \quad \text{or} \quad 3.85 \times 10^{-5} C$$

When $q_2 = 1.15 \times 10^{-5} C$; $q_1 = 3.85 \times 10^{-5} C$

and when,

$$q_2 = 3.85 \times 10^{-5} C \; ; \; q_1 = 1.15 \times 10^{-5} C$$

1c Three charges were positioned as shown in the figure below. If $Q_1 = Q_2 = 8\mu C$ and $d = 0.5m$, determine q if the electric field at p is zero.



Solution

$Q_1 = Q_2 = 8\mu C = 8\times10^{-6}C$

$d = 0.5m$

$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{1}{\sqrt{5}/2}$

$\theta = \sin^{-1} 2/\sqrt{5}$

$= 63.43^\circ$

Note,

$(Hyp)^2 = (2d)^2 + d^2$

$(Hyp)^2 = 5d^2$

$Hyp = \sqrt{5d^2} = d\sqrt{5}$

$= \sqrt{5}/2$

$E_1 = \frac{kQ_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(\sqrt{5}/2)^2} = 57600\ N/C$

$E_2 = \frac{kQ_1}{r^2} = \frac{9\times10^9\times8\times10^{-6}}{(\sqrt{5}/2)^2} = 57600\ N/C$

$E_3 = \frac{kQ_1}{r^2} = \frac{9\times10^9 \times q}{(1)^2} = 9\times10^9 q\ N/C$

Vector	θ	x-Component ($E\cos\theta$)	y-Component ($E\sin\theta$)
$E_1 = 57600 N/C$	63.43	$57600\cos 63.43 = 25764.48$	$57600\sin 63.43 = 51517.44$
$E_2 = 57600 N/C$	63.43	$-57600\cos 63.43 = -25764.48$	$57600\sin 63.43 = 51517.44$
$E_3 = 9\times10^9 q\ N/C$	90°	$9\times10^9 q\cos 90^\circ = 0$	$9\times10^9 q\sin 90^\circ = 9\times10^9 q$

Therefore,

$\Sigma E_x = 0$ $\Sigma E_y = 103034.88 + 9\times10^9 q$

The magnitude at $P = 0$ is,

$0 = \sqrt{0^2 + (103034.88 + 9\times10^9 q)^2}$

$0 = 103034.88 + 9\times10^9 q$

$q = \frac{103034.88}{-9\cdot10^9} = -11.45\times10^{-6}c$

$= -11.45\mu C$

2a. Distinguish between the terms electric field and electric field intensity.

Answer

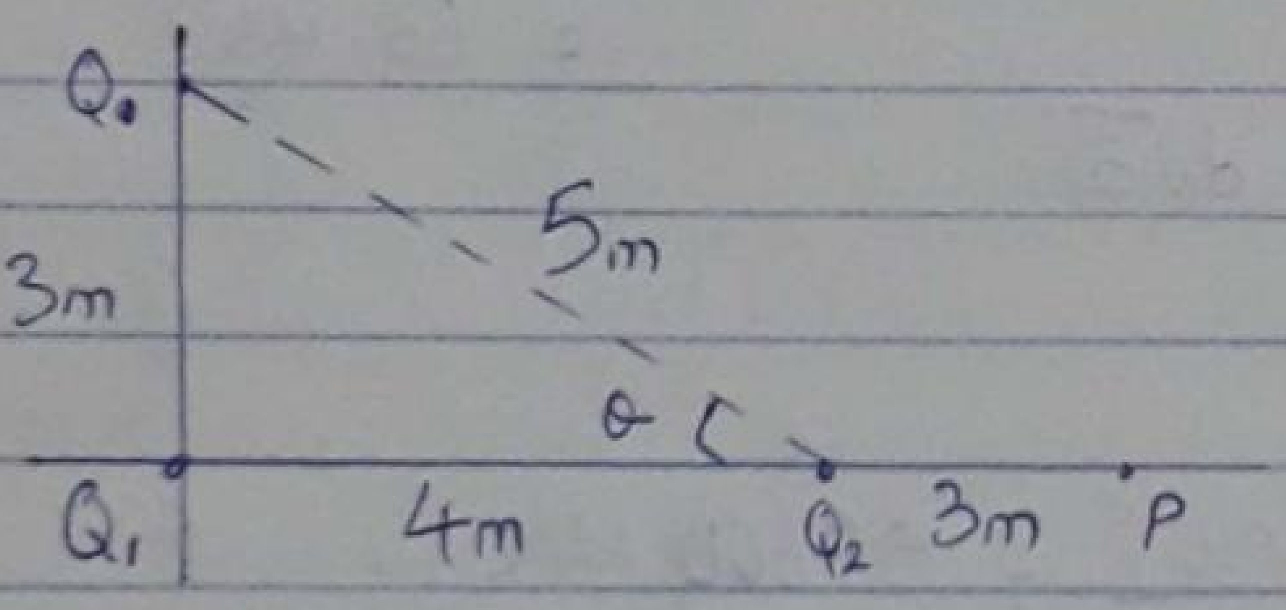
An electric field is a region of space which an electric charge will experience an electric force while electric field intensity is the force per unit charge of an electric field.

2b. A positive Charge $Q_1 = 8nC$ is at the origin, and a second positive Charge $Q_2 = 12nC$ is on the x-axis at $x = 4m$. Find;

(i) the net electric field at a point P on the x-axis at $x = 7m$.

(ii) the electric field at a point Q on the y-axis at $y = 3m$ due to the Charges.

Answer



$Q_1 = 8nC = 8 \times 10^{-9} C$

$Q_2 = 12nC = 12 \times 10^{-9} C$

$\theta = 36.87°$

(i) At point P,

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.4694 \, N/C$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 \, N/C$$

$$\therefore E_{net} = E_1 + E_2$$

$$E_{net} = 12 + 1.4694 = 13.4694 \, N/C$$

$$E_{net} \simeq 13.5 \, N/C$$

(ii) At point Q,

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \, N/C$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \, N/C$$

Vector	θ	$E_x = E\cos\theta$	$E_y = E\sin\theta$
$E_1 = 8\ N/C$	$90°$	$8\cos 90 = 0$	$8\sin 90 = 8$
$E_2 = 4.32\ N/C$	36.87	$4.32\cos 36.87 = 3.4171$	$4.32\sin 36.87 = 2.592$

$$\Sigma E_x = 3.4171 \quad \Sigma E_y = 10.592$$

$$\therefore E = \sqrt{\Sigma E_x^2 + \Sigma E_y^2}$$

$$E = \sqrt{(3.4171)^2 + (10.592)^2}$$

$$E = \sqrt{123.8671}$$

$$E = 11.13\ N/C$$

SECTION B

4a. What is Magnetic Flux?

Answer

Magnetic flux is defined as the strength of magnetic field represented by lines of force.

4b. An electron with a rest mass of 9.11×10^{-31} kg moves in a circular orbit of radius 1.4×10^{-7} m in a uniform magnetic field of 3.5×10^{-1} weber/meter square perpendicular to the speed with which electron moves. Find the cyclotron frequency of the moving electron.

Solution

$m = 9.11 \times 10^{-31}$ kg

$r = 1.4 \times 10^{-7}$ m

$B = 3.5 \times 10^{-1}$ weber/m^2

Cyclotron frequency, $\omega = \frac{qB}{m}$

$$= \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$\therefore \omega = 6.147 \times 10^{10}\ rads^{-1}$$

4c. Discuss your answer in 4b. above.

Answer

This simply implies that an electron mass 9.11×10^{-31} kg of Charge 1.6×10^{-19} C in motion in a magnetic field of 3.5×10^{-1} T is perpendicular with the field will have an angular frequency of 6.147×10^{10} rads^{-1}

5a. State Biot - Savart Law

Answer

1. The vector $\vec{dB}$ is perpendicular to $d\hat{l}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $\vec{dl}$ toward P.
2. The magnitude of $\vec{dB}$ is inversely proportional to r^2, where r is the distance from $\vec{dl}$ to P.
3. The magnitude of $\vec{dB}$ is proportional to the current I and the magnitude of the element $\vec{dl}$.
4. The magnitude of $\vec{dB}$ is proportional to $n\theta$, where θ is the angle between $\hat{r}$ and dl.

5b. Using the Biot - Savart Law, show that the magnitude of the magnetic field of a straight current carrying conductor is given as, $B = \frac{\mu_0 I}{2\pi r}$

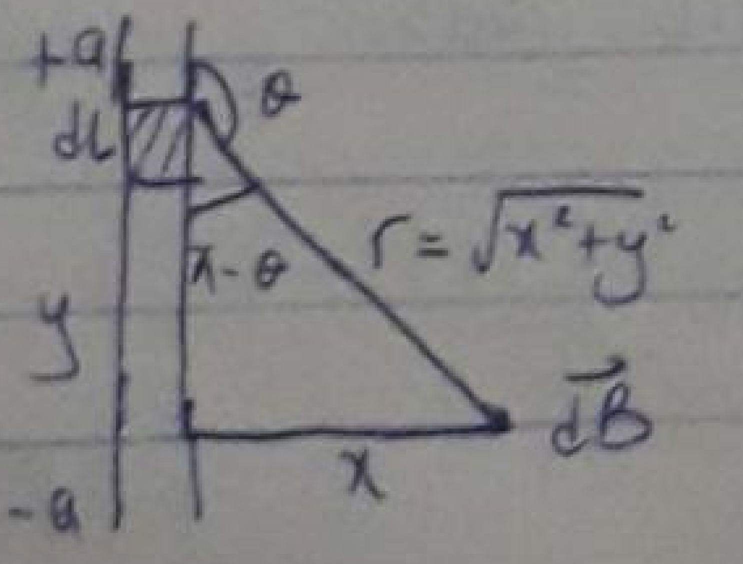
Solution

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \times \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$\int \vec{dB} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2} \qquad \left\{ \begin{aligned} \vec{dl} \times \hat{r} &= dlr \sin\theta \\ &= dl \sin\theta \end{aligned} \right\}$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$$



$$r = \sqrt{x^2 + y^2}$$

$$r^2 = x^2 + y^2$$

$$\theta = \pi - \theta$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\theta)}{r^2}$$

$$\sin(\pi-\theta) = x/r$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl}{r^2}\cdot\frac{x}{r}$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{dl}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{dl}{(x^2+y^2)^{3/2}}$$

$$dl \equiv dy$$

$$\therefore B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{1}{(x^2+y^2)^{3/2}}\,dy$$

$$B = \frac{\mu_0 I x}{4\pi}\,\frac{1}{x}\cdot\frac{y}{(x^2+y^2)^{3/2}}\Bigg|_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\cdot\frac{a}{x^2(x^2+a^2)^{1/2}} - \frac{-a}{x^2(x^2+a^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi}\cdot\frac{2a}{x^2(x^2+a^2)^{1/2}}$$

Also,

$$(x^2+a^2)^{1/2} \equiv a$$

$$a >> x$$

$$B = \frac{\mu_0 I}{4\pi x}\cdot\frac{2a}{a}$$

$$B = \frac{\mu_0 I}{2\pi x}, \quad x = r$$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$