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MECHATRONICS ENGINEERING
PHY 202

ASSIGNMENT

2(a) Electric field is a region of space in which an electric charge will experience an electric force.
Electric field intensity is the force per unit charge.

(b) $Q_1 = 8\mu c$
$Q_2 = 12\mu c$
$r = 4m$
$k = 8.99 \times 10^9 Nm^2/c^2$
$F = ?$
$E = ?$

$$E = \frac{F}{q_0}$$

Since $F = \frac{KQq_0}{r^2}$

$$E = \frac{\frac{KQq_0}{r^2}}{q_0} = \frac{KQ}{r^2} = \frac{8.99 \times 10^9 \times 8}{7^2} = \cancel{10.3 \times 10^9}$$
$$= \cancel{1.03 \times 10^9 N/C}$$
$$= 1.47 \times 10^9 N/c$$

$$E = \frac{KQ_2}{r^2} = \frac{8.99 \times 10^9 \times 12}{3^2} = 11.99 \times 10^9 = 1.2 \times 10^{10} N/c$$

3(a) i. Volume charge density $\rightarrow \rho = \frac{dQ}{dV} \Rightarrow dQ = \rho dV$

ii. Surface charge density $\rightarrow \sigma = \frac{dQ}{dA} \Rightarrow dQ = \sigma dA$

iii. Linear charge density $\rightarrow \lambda = \frac{dQ}{dL} \Rightarrow dQ = \lambda dl$

(b) The electric potential difference between two points in an electric field can be defined as the workdone per unit charge against electric forces when a charge is transported from one point to another.

It's measured in volts or Joules per coulomb (JC^{-1})

To move a test charge from A to B at constant velocity, an external force of $F = -q_0 E$ must act on the charge.

Therefore, the elemental workdone dW is given as;

$$dW = F \cdot dl \text{ --- (i)}$$

Since $F = -q_0 E$ --- (ii)

$$\therefore dW = -q_0 E \cdot dl \text{ --- (iii)}$$

Total workdone in moving the test charge from A to B is;

$$W(A \to B)_{Ag} = -q_0 \int_A^B E\, dl \text{ --- (iv)}$$

From the definition of electric p.d;

$$V_B - V_A = \frac{W(A \to B)_{Ag}}{q_0} \text{ --- (v)}$$

$$\therefore V_B - V_A = -\int_A^B E\, dl$$

(c) $V_P = \frac{1}{4\pi\varepsilon_0} \times \left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right]$

$Q_1 = 10\mu C$

$Q_2 = -2\mu C$

$r_1 = 0m$

$r_2 = 4m$

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2/C$

$$V_P = 9\times10^9 \times \left[\frac{10}{0} + \left(\frac{-2}{4}\right)\right]$$

$$= 9\times10^9 \times [-0.5]$$

$$= -4.5\times10^9 V$$

4(a) Magnetic flux are imaginary lines along which a free north pole would tend to move if placed in the field.

(b) $m = 9.11 \times 10^{-31}$ kg

$r = 1.4 \times 10^{-7}$ m

$B = 3.5 \times 10^{-1}$ Weber/metre

$q = 1.6 \times 10^{-19}$ C

$\omega = ?$ $\quad V = ?$

$$\cancel{\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = 0.615 \times 10^{-19-1+31}}$$

$$\cancel{= 0.615 \times 10^{11}}$$

$$\cancel{= 6.15 \times 10^{10} \text{ rads}^{-1}}$$

Since the proton moves in a circular orbit;

$$F_B = qvB = \frac{mv^2}{r}$$

$$m_e V = qBr$$

$$v = \frac{qBr}{m_e}$$

$$= \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1} \times 1.4 \times 10^{-7}}{9.11 \times 10^{-31}}$$

$$= 0.861 \times 10^{-19-1-7+31}$$

$$= 0.861 \times 10^{4}$$

$$= 8.61 \times 10^{3} \text{ ms}^{-1}$$

~~we~~ therefore $\omega = \frac{qB}{m_e} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$

$$= 0.615 \times 10^{11}$$

$$= 6.15 \times 10^{10} \text{ rad/s}$$

5(a) Biot-Savart law is based on the following observation for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{l}$ of a wire carrying a steady current I.

(b) $d\vec{B} = \frac{\mu_0}{4\pi} \times \frac{I\,d\vec{l} \times \hat{r}}{r^2}$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0}{4\pi}\int \frac{I\,dl \sin\theta}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

considering $r^2 = x^2 + y^2$ (pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2+y^2}$$

But $\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}}\,dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}}\,dy$$

Using special integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. i.e when a is much larger than x; $(x^2+a^2)^{1/2} \cong a$, as $a \to \infty$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$