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MAT102

① if $A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$
find $-8(A+B) \cdot (C-A)$

Solu:

$$-8(A+B) = -8\begin{pmatrix} 5i - 7j - 6k \\ \quad\; j + 4k \end{pmatrix}$$

$$-8(5i - 6j - 2k)$$

$\therefore -8(A+B) = -40i + 48j + 16k$

$$(C-A) = \begin{array}{r} 9i - 4j + k \\ \underline{5i - 7j - 6k} \\ 4i - 13j + 7k \end{array}$$

$$-8(A+B) \cdot (C-A) = \begin{array}{r} -40i + 48j + 16k \\ \underline{4i - 13j + 7k} \\ -160i - 624j + 112k \end{array}$$

2. $x = -3t$, $y = t^2$, $z = 4t^3$ where $t = 1$

$r = -3ti + t^2j + 4t^3k$

$\frac{dr}{dt} = -3i + 2tj + 12t^2$

at $t=1$

$\frac{dr}{dt} = -3i + 2j + 12k$

$\left|\frac{dr}{dt}\right| = \sqrt{(-3)^2 + (2)^2 + (12)^2} = \sqrt{9 + 4 + 144}$

$= \sqrt{157} = 12.53$

Hence $T = \frac{-3 + 2j + 12k}{12.53}$

3 $x = -8t^2,\ y = t^2 - 4t,\ z = t+1$ where $t=1$

$r = -8t^2 i + (t^2 - 4t)j + (t+1)k$

$\frac{dr}{dt} = 16ti + (2t-4)j + tk$

Where $t=1$ $= 16ti + (2t-4)j + 1k$

4) $A = i + 2j - 4k,\ B = 2i - 3j + 1k,\ C = 6j - 3k$ find $(A \times B) \times C$

Soln

$$A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 2 & -3 & 1 \end{vmatrix}$$

$$i\begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$i(2-12) - j(1-(-8)) + 1k(-3-4)$

$= -10i + 9j - 7k$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -10 & 9 & -7 \\ 0 & 4 & -3 \end{vmatrix}$$

$$= i \begin{vmatrix} 9 & -7 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -10 & -7 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} -10 & 9 \\ 0 & 4 \end{vmatrix}$$

$= i\left(-27 - (-28)\right) - j\left(30 - 0\right) + k\left(-40 - 0\right)$

$= i(1) - j(30) + k(-40)$

$= 1i - 30j - 40k$

5) $R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$

Soln.

$\int R\,dt = \int (4\sin 3t\, i + 4e^{3t} j + 7t^3 k)\,dt$

$= \int 4\sin 3t\, i\, dt + \int 4e^{3t} j\, dt + \int 7t^3 k\, dt$

$\int R\,dt = \frac{-4}{3} \cos 3t\, i + \frac{4}{3} e^{86} + \frac{7}{4} t^3 + C$