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18/ENG02/084

4 $A = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix}$

$R_2 - 4R_1 = 4\ 0\ 2$
$-4\ -12\ 24$
$0\ -12\ 22$

$A = \begin{pmatrix} 1 & -3 & 6 \\ 0 & 12 & -22 \\ 8 & 5 & 1 \end{pmatrix}$

$R_3 - 8R_1 = 8\ 5\ 1$
$-8\ -24\ 48$
$0\ -29\ 47$

$A = \begin{pmatrix} 1 & -3 & 6 \\ 0 & 12 & -22 \\ 0 & -29 & 47 \end{pmatrix}$

Divide the 2nd row by 12

$A = \begin{pmatrix} 1 & -3 & 6 \\ 0 & 1 & -11/6 \\ 0 & 29 & -47 \end{pmatrix}$

multiply the 2nd row by 29

$A = \begin{pmatrix} 1 & -3 & 6 \\ 0 & 29 & -319/6 \\ 0 & 29 & -47 \end{pmatrix}$

Subtract the 2nd row from the 3rd row

$A = \begin{pmatrix} 1 & -3 & 6 \\ 0 & 1 & -11/6 \\ 0 & 0 & 37/6 \end{pmatrix}$

$A = \begin{pmatrix} 1 & -3 & 6 \\ 0 & 12 & -22 \\ 0 & 0 & 37/6 \end{pmatrix}$

∴ Matrix rank is 3

2. $B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{pmatrix}$ $B^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \end{pmatrix}$

$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 & 0 \\ 0 & -3 & 3 \\ 2 & 1 & 4 \end{pmatrix}$

multiply row 1 by 2.

$2R_1$

$B^T = \begin{pmatrix} 2 & 2 & 0 \\ 0 & -3 & 3 \\ 2 & 1 & -4 \end{pmatrix}$

Subtract 1st row from the 3rd Row and restore it.

$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -3 & 3 \\ 0 & -1 & -4 \end{pmatrix}$

Divide 2nd Row by -3.

$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & -4 \end{pmatrix}$

$-1(R_2) \Rightarrow$ multiply by -1.

$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & -4 \end{pmatrix}$

$\Rightarrow R_1 - R_3$ and restore it.

$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -5 \end{pmatrix}$

restore R_2 to original value

$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -5 \end{pmatrix}$

$\therefore$ Matrix rank is 3.

3 $A+C = \begin{bmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 9 \\ 10 & -7 & 3 \\ 9 & 2 & 3 \end{bmatrix}$

$(A+C)^T = \begin{bmatrix} 1 & 10 & 9 \\ 1 & -7 & 2 \\ 9 & 3 & 3 \end{bmatrix}$

$R_1 - R_2$

$(A+C)^T = \begin{bmatrix} 1 & 10 & 9 \\ 0 & -17 & -7 \\ 9 & 3 & 3 \end{bmatrix}$

$9R_1$

$(A+C)^T = \begin{bmatrix} 9 & 90 & 81 \\ 0 & -17 & -7 \\ 9 & 3 & 3 \end{bmatrix}$

$R_1 - R_3$

$(A+C)^T = \begin{bmatrix} 1 & 10 & 9 \\ 0 & -17 & -7 \\ 0 & -87 & -78 \end{bmatrix}$

Divide the 2nd row by -17

$(A+C)^T = \begin{bmatrix} 1 & 10 & 9 \\ 0 & 1 & 7/17 \\ 0 & -87 & -78 \end{bmatrix}$

mulliply the 2nd row by -87.

$(A+C)^T = \begin{bmatrix} 1 & 10 & 9 \\ 0 & -87 & -609/17 \\ 0 & -87 & -78 \end{bmatrix}$

Subtract the R_2 row from the 3rd row

$(A+C)^T = \begin{bmatrix} 1 & 10 & 9 \\ 0 & -17 & -7 \\ 0 & 0 & -717/17 \end{bmatrix}$

∴ Matrix rank is 3

4 $B+C = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 & 5 \\ 7 & -9 & 2 \\ 1 & 0 & -2 \end{bmatrix}$

$R_2 - 7R_1$

$= \begin{bmatrix} 1 & 5 & 5 \\ 0 & -44 & -33 \\ 1 & 0 & -2 \end{bmatrix}$

$R_1 - R_3$

$= \begin{bmatrix} 1 & 5 & 5 \\ 0 & -44 & -33 \\ 0 & -5 & -7 \end{bmatrix}$

$R_3/-44$ (Divide R_2 by -44)

$= \begin{bmatrix} 1 & 5 & 5 \\ 0 & 1 & 3/4 \\ 0 & -5 & -7 \end{bmatrix}$

$-5R_2$

$= \begin{bmatrix} 1 & 5 & 5 \\ 0 & -5 & -15/4 \\ 0 & -5 & -7 \end{bmatrix}$

$R_2 - R_3$

$= \begin{bmatrix} 1 & 5 & 5 \\ 0 & 1 & 3/4 \\ 0 & 0 & -13/4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 & 5 \\ 0 & -44 & -33 \\ 0 & 0 & -13/4 \end{bmatrix}$

∴ Matrix rank is 3

5 $(A+B+C) = \begin{bmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 5 \\ 7 & -9 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & 11 \\ 11 & -9 & 4 \\ 9 & 5 & -1 \end{bmatrix}$

Divide the 1st row by 2

$= \begin{bmatrix} 1 & -1 & 11/2 \\ 11 & -9 & 4 \\ 9 & 5 & -1 \end{bmatrix}$

$\underline{11R_1}$

$= \begin{bmatrix} 11 & -11 & 121/2 \\ 11 & -9 & 4 \\ 9 & 5 & -1 \end{bmatrix}$

$\underline{11R_1 - R_2}$

$= \begin{bmatrix} 4 & -1 & 11/2 \\ 0 & 2 & -113/2 \\ 9 & 5 & -1 \end{bmatrix}$

$\underline{9(R_1) - R_3}$

$= \begin{bmatrix} 1 & -1 & 11/2 \\ 0 & 2 & -113/2 \\ 0 & 14 & -101/2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & 11 \\ 0 & 2 & -113/2 \\ 0 & 14 & -101/2 \end{bmatrix}$

Divide Row(2) by 2

$= \begin{bmatrix} 2 & -2 & 11 \\ 0 & 1 & -113/4 \\ 0 & 14 & -101/2 \end{bmatrix}$

multiply 2nd row by 14

$= \begin{bmatrix} 2 & -2 & 11 \\ 0 & 14 & -791/2 \\ 0 & 14 & -101/2 \end{bmatrix}$

$R_2 - R_3$

$= \begin{bmatrix} 2 & -2 & 11 \\ 0 & 1 & -113/4 \\ 0 & 0 & 345 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & 11 \\ 0 & 2 & -113/2 \\ 0 & 0 & 345 \end{bmatrix}$

∴ Matrix rank is 3.