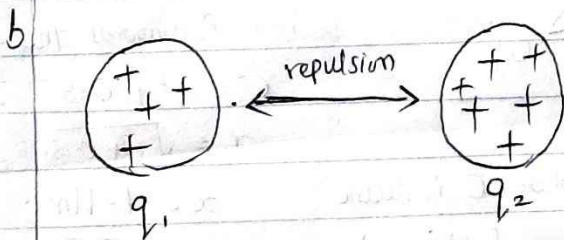
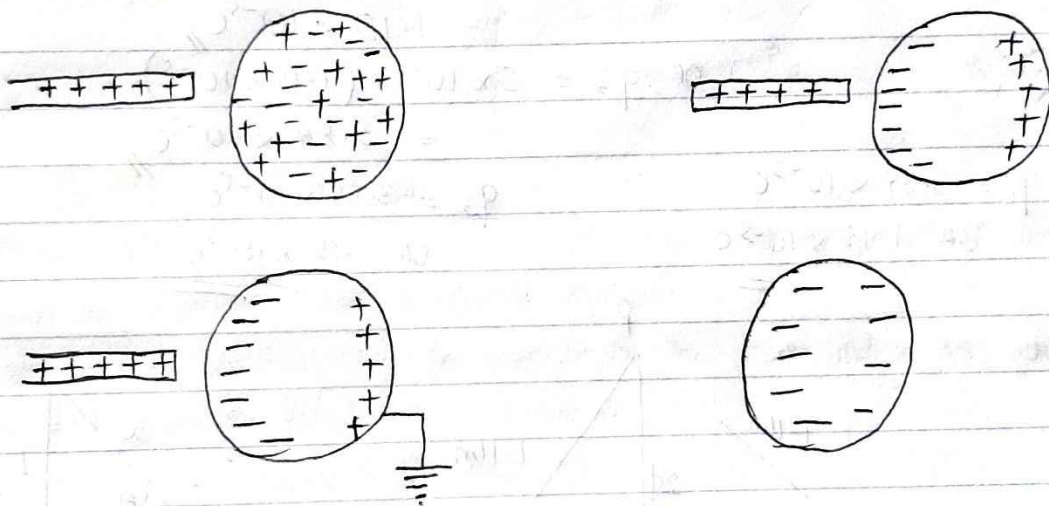


SECTION A

QUESTION 1

a Charging by Induction

If a ~~positively~~ ^{positively} charged rubber rod is brought near a neutral conducting sphere that is insulated with no conducting path. The repulsive force between the protons on the rod and those on the sphere, causes a redistribution where the protons move to the farthest ^{opposite} side of the sphere. The region close to the rubber rod possesses ~~positive~~ ^{negative} charges. If a grounded connecting wire is connected to the sphere, some if not all the protons leave the sphere to the earth. When the rubber rod is finally removed, the excess negative charges which were induced remain on the sphere making it negatively charged.



$$q_1 + q_2 = 5 \times 10^{-5} \text{ --- (1)}$$

Force, $F = 1\text{N}$

distance apart, $d = 2\text{m}$

$q_1 = ?$ $q_2 = ?$

$K = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

But $F = \frac{Kq_1q_2}{d^2}$

$$1 = \frac{9 \times 10^9 \times q_1q_2}{(2)^2}$$

$$2^2 = 9 \times 10^9 q_1q_2$$

$$q_1q_2 = \frac{2^2}{9 \times 10^9}$$

$$q_1 q_2 = 4.44 \times 10^{-10}$$

but from eqn ① $q_1 + q_2 = 5 \times 10^{-5}$

$$\therefore q_2 = 5 \times 10^{-5} - q_1 \quad \text{--- ②}$$

Put ② into ①

$$q_1 (5 \times 10^{-5} - q_1) = 4.44 \times 10^{-10}$$

$$5 \times 10^{-5} q_1 - q_1^2 = 4.44 \times 10^{-10}$$

$$\therefore q_1^2 - 5 \times 10^{-5} q_1 + 4.44 \times 10^{-10} = 0$$

$$\therefore q_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q_1 = \frac{-(-5 \times 10^{-5}) \pm \sqrt{(-5 \times 10^{-5})^2 - 4(1)(4.44 \times 10^{-10})}}{2(1)}$$

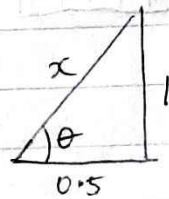
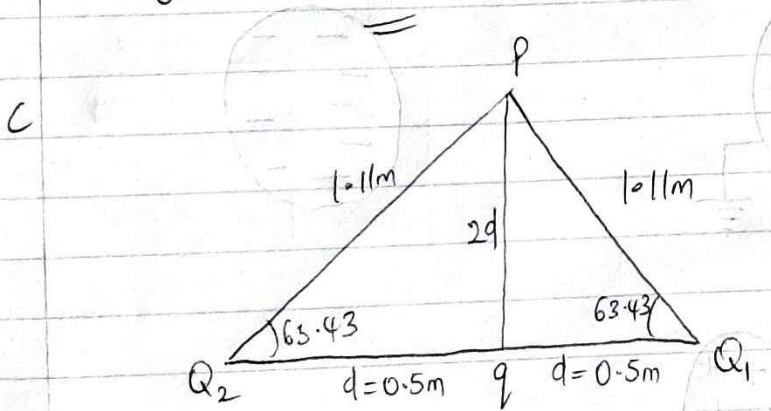
$$q_1 = 3.84 \times 10^{-5} \text{ C or } 1.15 \times 10^{-5} \text{ C}$$

In eqn ② $q_2 = 5 \times 10^{-5} - (3.84 \times 10^{-5}) = 1.15 \times 10^{-5} \text{ C}$

OR $q_2 = 5 \times 10^{-5} - (1.15 \times 10^{-5}) = 3.84 \times 10^{-5} \text{ C}$

$\therefore q_1 = 3.84 \times 10^{-5} \text{ C}$
OR $1.15 \times 10^{-5} \text{ C}$

$q_2 = 3.84 \times 10^{-5} \text{ C}$
OR $1.15 \times 10^{-5} \text{ C}$



From Pythagoras theorem

$$x^2 = 1^2 + 0.5^2$$

$$x = \sqrt{1^2 + 0.5^2}$$

$$x = 1.1 \text{ m}$$

$$\tan \theta = \frac{1}{0.5}$$

$$\therefore \theta = \tan^{-1}(1/0.5) = 63.43^\circ$$

For the charge at P to be zero,

$$E_{\text{net}} = 0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_q \quad \text{where } E \text{ is electric field intensity}$$

$$E_2 = E_1 = \frac{kq_1}{(1.1)^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.1)^2} = 5.84 \times 10^4$$

$$E_q = \frac{kq}{(2d)^2} = \frac{9 \times 10^9 \times q}{(1)^2} = 9 \times 10^9 q$$

Electric Field Intensity	Angle (θ)	Horizontal Component (x)	Vertical component (y)	
E_1	5.84×10^4	63.43	$5.84 \times 10^4 \times (-\cos 63.43)$ $= -2.61 \times 10^4$	$5.84 \times 10^4 \times \sin 63.43$ $= 5.22 \times 10^4$
E_2	5.84×10^4	63.43	$5.84 \times 10^4 \times \cos 63.43$ $= 2.61 \times 10^4$	$5.84 \times 10^4 \times \sin 63.43$ $= 5.22 \times 10^4$
E_q	$9 \times 10^9 q$	90°	$9 \times 10^9 q \times \cos 90 = 0$ $\Sigma_x = 0$	$9 \times 10^9 q \times \sin 90 = 9 \times 10^9 q$ $\Sigma_y = 10.44 \times 10^4 + 9 \times 10^9 q$

$$\Sigma_{net} = 0 = \sqrt{(\Sigma_x)^2 + (\Sigma_y)^2}$$

$$0 = \sqrt{0^2 + (10.44 \times 10^4 + 9 \times 10^9 q)^2}$$

$$0 = 10.44 \times 10^4 + 9 \times 10^9 q$$

$$-10.44 \times 10^4 = 9 \times 10^9 q$$

$$\therefore q = \frac{-10.44 \times 10^4}{9 \times 10^9}$$

$$q = -1.16 \times 10^{-5} \text{C}$$

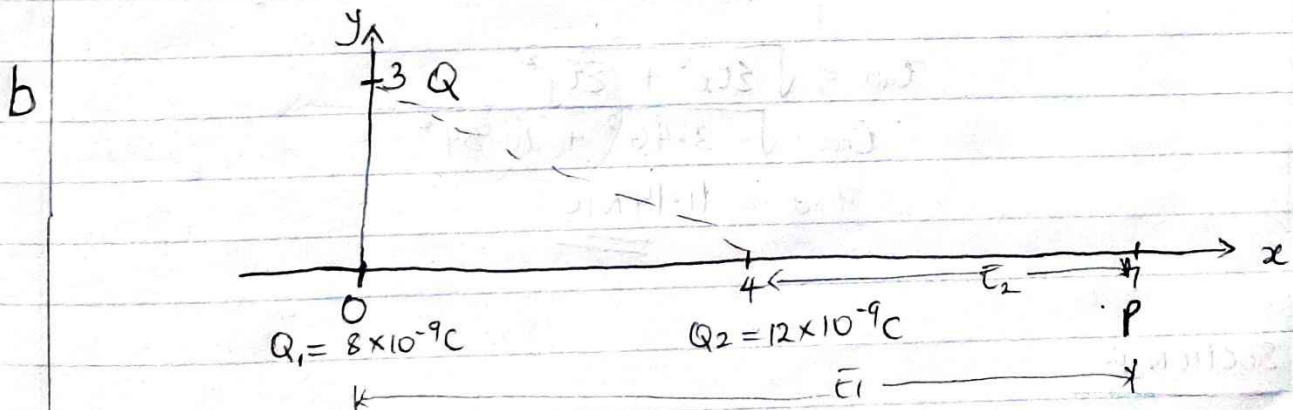
$$q = -11.6 \times 10^{-6} \text{C} = -11.6 \mu\text{C}$$

QUESTION 2

a Electric field can be said to be the region of space where the influence of electricity can be felt or detected

Electric field intensity can be said to be the force acting per unit charge in the field (ie the force on every charge).

Mathematically $E = \frac{F}{q}$



i Net electric field at P

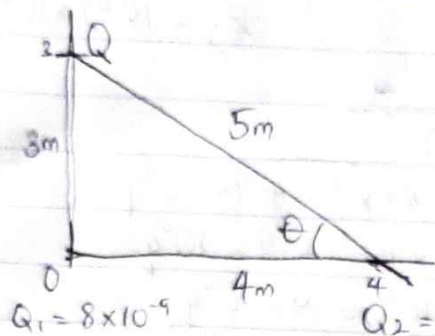
$$E_{net} = \vec{E}_1 + \vec{E}_2$$

where $E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2}$

$$= 1.47 \text{N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 \text{ N/C}$$

$$\begin{aligned} \vec{E} &= E_1 + E_2 \\ &= 1.47 + 12 \\ &= \underline{\underline{13.47 \text{ N/C}}} \end{aligned}$$



$$\tan \theta = \frac{3}{4}$$

$$\begin{aligned} \theta &= \tan^{-1}(3/4) \\ &= \underline{\underline{36.87^\circ}} \end{aligned}$$

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

Vector	Angle θ	Horizontal Component (x)	Vertical Component (y)
$E_1 = 8 \text{ N/C}$	90	$8 \times \cos 90 = 0$	$8 \times \sin 90 = 8$
$E_2 = 4.32 \text{ N/C}$	36.87	$4.32 \times \cos 36.87 = 3.46$	$4.32 \times \sin 36.87 = 2.59$
		$\Sigma E_x = 3.46 \text{ N/C}$	$\Sigma E_y = 10.59 \text{ N/C}$

$$\begin{aligned} E_{\text{net}} &= \sqrt{\Sigma E_x^2 + \Sigma E_y^2} \\ E_{\text{net}} &= \sqrt{3.46^2 + 10.59^2} \\ E_{\text{net}} &= \underline{\underline{11.14 \text{ N/C}}} \end{aligned}$$

SECTION B

4a Magnetic flux is defined as the strength of magnetic field represented by the lines of force

b mass of electron, $m = 9.11 \times 10^{-31} \text{ kg}$
radius, $r = 1.4 \times 10^{-7} \text{ m}$

Charge of electron, $q = 1.6 \times 10^{-19} \text{ C}$
 $B = 3.5 \times 10^{-1} \text{ W/m}^2$

Cyclotron frequency = angular speed = $\omega = \frac{qB}{m}$

$$\omega = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$\omega = \underline{\underline{6.15 \times 10^{10} \text{ rad/s}}}$$

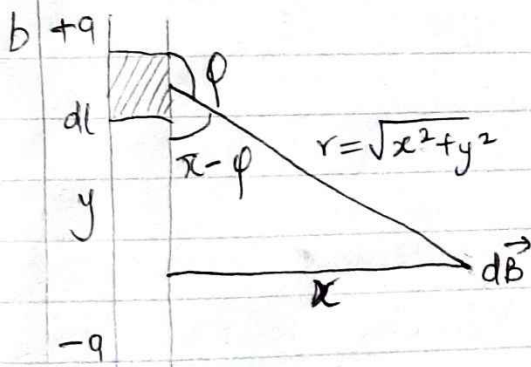
c The answer (cyclotron frequency) means that the charged particles (cyclotrons) circulate round the orbit at the speed of 6.15×10^{10} radians in one second.

5a The Biot-Savart law states that the magnitude of the magnetic field, B is given as

$$B = \frac{\mu_0}{4\pi} \int \frac{Idl \sin \theta}{r^2}$$

where μ_0 is permeability of free space

θ is the angle
 r is the distance.



From Biot-Savart's Law

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

But $\sin(\pi - \phi) = \sin \theta$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

From the diagram $r^2 = x^2 + y^2$ (Pythagoras)

$$\sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

$$\text{If } dl = dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy$$

$$\text{By special integrals } \int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

$$\therefore B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{2\pi x} \left(\frac{a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

as $a \rightarrow \infty$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi x}}$$