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MECHATRONICS ENGINEERING April 8th 2020

19/ENG05/017

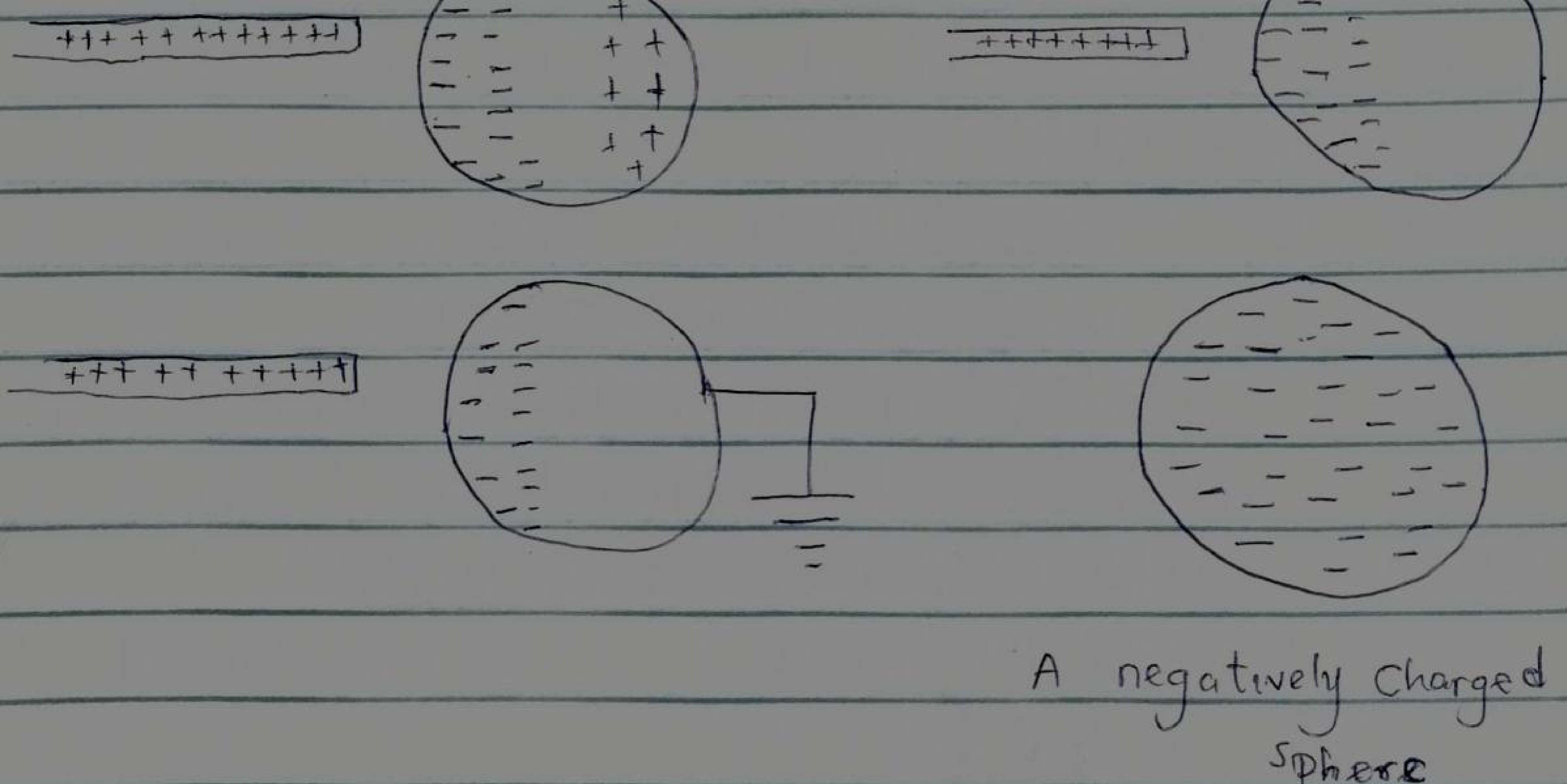
SECTION A

1. (a) Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

Answer

Consider a positively charged rubber rod brought near a neutral (uncharged) Conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to the positively charged rod has an excess of negative charge because of the migration of protons away from the location. If a grounded conducting wire is then connected to the sphere, some of the protons leave the conducting sphere and travel to the earth. If the wire to ground is then removed, the conducting sphere is left with an excess of induced negative charge.

finally, when the rubber rod is removed from the vicinity of the sphere, the induced negative charge remains on the unground sphere and becomes Uniformly distributed Over the surface of the sphere.



A negatively Charged sphere

1(b) Each of two small sphere is charged positively, the combined charge being $5.0\times10^{-5}C$. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, calculate the charge on each sphere.

Solution

$q_1 + q_2 = 5.0\times10^{-5}C$

$f = 1.0N$

$r = 2.0m$

q_1 and q_2 (+vely charged)

using Coulomb's law;

$$f = \frac{kq_1q_2}{r^2}$$

$k = 9.0\times10^{9}Nm^2/C^2$

$$1.0 = \frac{9.0\times10^{9}\times q_1 q_2}{(2.0)^2}$$

$$4.0 = 9.0\times10^{9}\times q_1q_2$$

$$q_1q_2 = \frac{4.0}{9.0\times10^{9}}$$

$$q_1q_2 = 4.44\times10^{-10}C^2 \text{ — eqn II}$$

$$q_1 + q_2 = 5.0\times10^{-5}C \text{ — eqn(I)}$$

$$q_1 = \frac{4.4\times10^{-10}}{q_2} \text{ — eqn(III)}$$

Substitute eqn III into I

$$\frac{4.4\times10^{-10}}{q_2} + \frac{q_2}{1} = 5.0\times10^{-5}$$

$$q_2^2 + 4.4\times10^{-10} - 5.0\times10^{-5} = 0$$

using quadratic formula; $q_2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$q_2 = \frac{-(-5.0\times10^{-5})\pm\sqrt{(-5.0\times10^{-5})^2-4(4.4\times10^{-10})}}{2}$$

$$q_2 = \frac{5.0\times10^{-5}\pm\sqrt{7.4\times10^{-10}}}{2}$$

$$q_2 = \frac{5\times10^{-5} \pm 2.72\times10^{-5}}{2}$$

$q_2 = 3.86\times10^{-5}$ or $q_2 = 1.15\times10^{-5}$

Imput the value of q_2 into eqn iii

$$q_1 = \frac{4.4\times10^{-10}}{3.86\times10^{-5}} \qquad q_1 = \frac{4.4\times10^{-10}}{1.15\times10^{-5}}$$

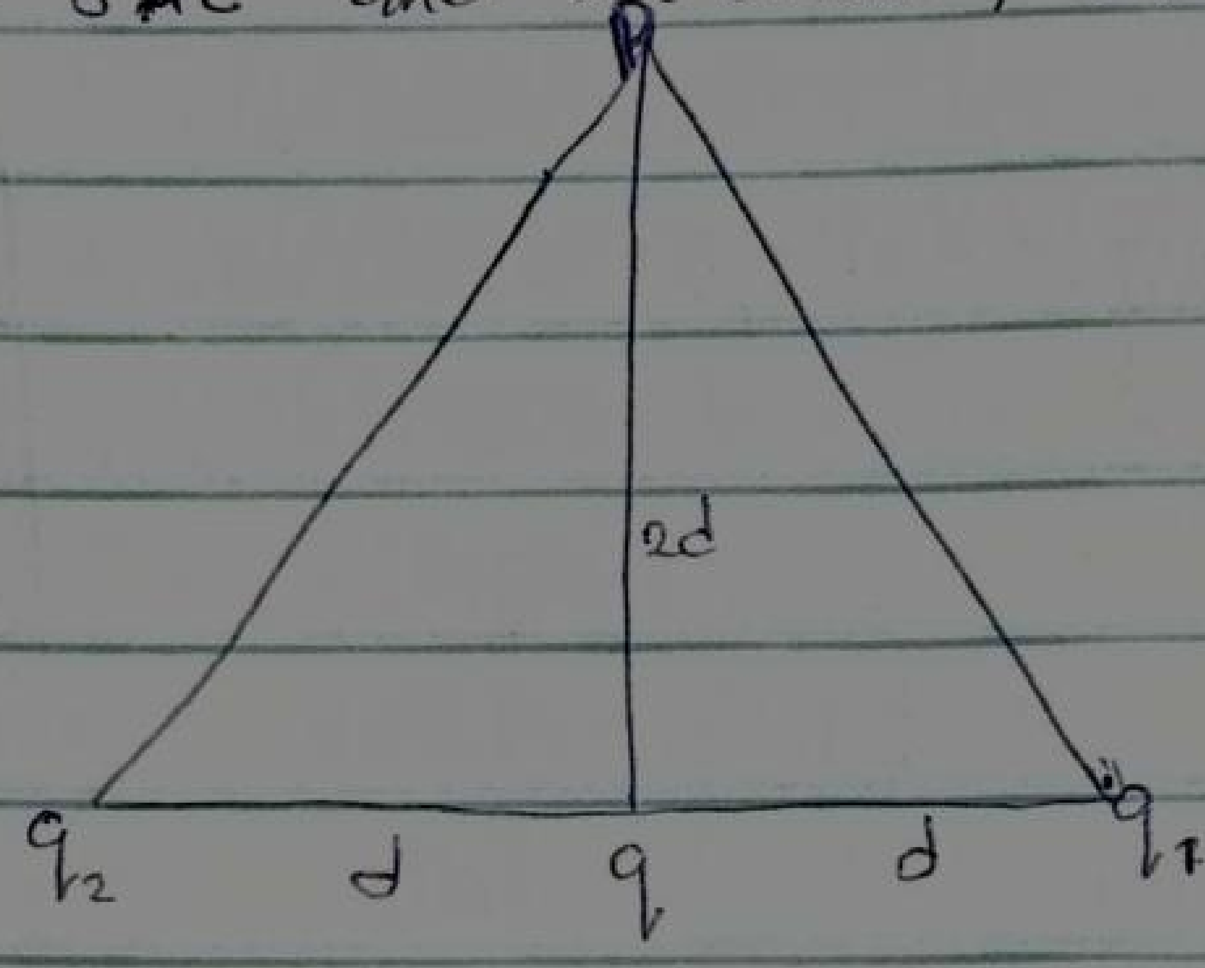
$q_1 = 1.134\times10^{-5}$ $\qquad$ $q_1 = 3.826\times10^{-5}$

The charge on each sphere

$\therefore$ $q_1 = 1.134\times10^{-5}$ or 3.826×10^{-5}

$q_2 = 3.86\times10^{-5}$ or 1.15×10^{-5}

(c) Three charges were positioned as shown below in figure below. If $Q_1 = Q_2 = 8\mu C$ and $d = 0.5m$, determine q if the electric field at P is Zero.



Solution

To find the distance between q_1, q_2 and P

Using Pythagora's theorem; $c^2 = a^2 + b^2$

~~$r^2 = d^2 + (2d)^2$~~ $\qquad$ $r^2 = d^2 + (2d)^2$

~~$r = \sqrt{0.5^2 + 4(0.5)^2}$~~ $\qquad$ $r = \sqrt{0.5^2 + 4(0.25)}$

~~$r = \sqrt{0.25 + 2}$~~ $\qquad$ $r = \sqrt{0.25 + 1}$

~~$r = \sqrt{2.25}$~~ $\qquad$ $r = \sqrt{1.25}$

~~$r = 0.8660$ 1.5~~ $\qquad$ $r = 1.118$

~~$\sin\theta = \frac{2d}{}$~~ $\qquad$ $\theta = \sin^{-1}\frac{1}{1.118} = \theta = 63.43^0$

~~$\theta = \sin^{-1} 2d = \sin^{-1} \frac{1}{}$~~ $\qquad$ ~~$= \sin^{-1} 1.1547$~~

~~$\theta = \sin^{-1}\frac{1}{1.5}$ $\quad$ $\theta = \sin^{-1} 0.667$ 0.8660~~

~~$\theta =$~~

To find E_1 E_2 and E_0

$$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(\sqrt{1.25})^2} = 57600\ N/C$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(\sqrt{1.25})^2} = 57600\ N/C$$

$$E_0 = \frac{kq}{r^2} = \frac{9\times10^9 \times q}{1} = 9\times10^9 q\ N/C$$

vectors	Angles	X - component	Y - component
57600	63.43^0	57600 cos 63.432 = 25763.95	57600 sin 63.43 = 51516.78
57600	63.43^0	-25763.95	51516.781
$9\times10^9 q$	0^0	0	$9\times10^9 q$
		$\sum_x = 0$	$\sum_y = 103033.56 + 9\times10^9 q$

$$E = \sqrt{\Sigma_x^2 + \Sigma_y^2}$$

$$E^2 = \Sigma_x^2 + \Sigma_y^2$$

At P_j: $E = 0$

$$0^2 = 0^2 + (103033.56 + 9\times10^9 q)^2$$

$$0 = 0 + 1.062\times10^{10} + 8.1\times10^{19} q^2$$

$$q^2 = \frac{1.062\times10^{10}}{8.1\times10^{19}}$$

$$q = \sqrt{1.311\times10^{-10}}$$

$$q = 1.1449\times10^{-5}$$

$$q \approx 11.45\ \mu C$$

2 (a) Distinguish between the terms; electric field and electric field intensity.

Answer

Electric field is a region in space where an electric charge will experience an electric force.

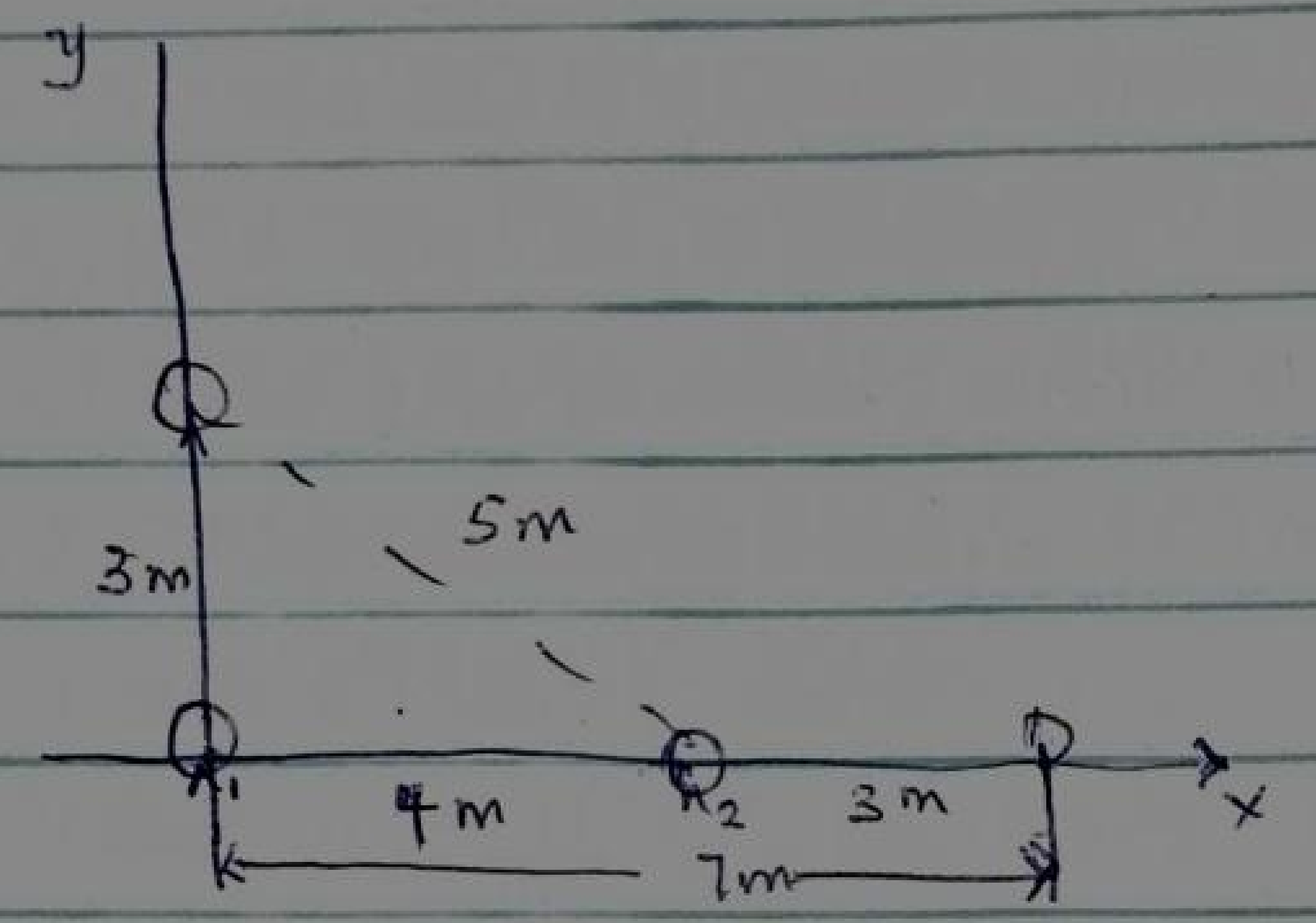
Electric field intensity is force per unit charge.

(b) A positive charge $Q_1 = 8\,nC$ is at the Origin and a second positive charge $Q_2 = 12\,nC$ is on the x-axis at $x = 4\,m$. find

i) the net electric field at a point P on the x-axis at $x = 7\,m$

ii) the electric field at a point Q on the y axis at $y = 3\,m$ due to the charges.

Solution



i) Net Electric field at a point P

E_1 and E_2

$$E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^{9}\times 8\times10^{-9}}{7^2} = \cancel{1.469387\,N/C}\ \ 1.47\times10^{7}\,N/C$$

$$E_2 = \frac{KQ_2}{r^2} = \frac{9\times10^{9}\times 12\times10^{-9}}{9} = 12\,N/C$$

$$E_{Net} = 12 + 1.47\times10^{7} = 13.47\times10^{7}\,N/C$$

ii) Electric field at a point Q

The distance Q to Q_2

$3^2 + 4^2 = r^2$

$\sqrt{25} = r$

$r = 5m$

$\sin\theta = \frac{3}{5}$

$\theta = \sin^{-1} 3/5$

$\theta = 36.87°$

To find E_1 and E_2

$E_1 = \frac{KQ_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-9}}{3^2} = 8\ N/C$

$E_2 = \frac{KQ_2}{r} = \frac{9\times10^9 \times 12\times10^{-9}}{5^2} = 4.32\ N/C$

vector	Angle	x-component	Y-component
8	90°	0	8
4.32	36.87	3.456	2.592
		$\Sigma_x = 3.456$	$\Sigma_y = 10.592$

$E = \sqrt{\Sigma_x^2 + \Sigma_y^2}$

$E = \sqrt{(3.456)^2 + (10.592)^2}$

$E = 11.142$

$\tan\theta = \frac{10.592}{3.456}$

$\theta = \tan^{-1} 3.0648$

$\theta = 71.93°$

SECTION B

4(a) What is Magnetic flux?

Magnetic flux is the strength of magnetic field represented by lines of force. It is usually represented by the symbol ϕ.

(b) An electron with a rest mass of 9.11×10^{-31} kg moves in a circular orbit of radius 1.4×10^{-7}m in a uniform magnetic field of 3.5×10^{-1} weber/meter square Perpendicular to the speed with which electron moves. find the cyclotron frequency of the moving electron.

solution

$m = 9.11 \times 10^{-31}$ kg

$r = 1.4 \times 10^{-7}$ m

$B = 3.5 \times 10^{-1}$ weber/meter square

$q = 1.6 \times 10^{-19}$ C

$w = ?$

$v = ?$

using;

$$w = \frac{qB}{m}$$

$$w = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$w = 6.1471 \times 10^{10} \text{ rads}^{-1}$$

$$w = \frac{v}{r}$$

$$v = 6.1471 \times 10^{10} \times 1.4 \times 10^{-7}$$

$$v = 8605.94 \text{ m/s}$$

(c) Discuss your answer in 4b above

The cyclotron frequency ~~6.1471×10^{10} rads^{-1}~~ of an electron of mass 9.11×10^{-31} kg and Charge 1.6×10^{-19}C moving ~~in motion~~ with a speed of 8605.94m/s and perpendicular to a uniform magnetic field of 3.5×10^{-1} weber/meter square is 6.1471×10^{10} rads^{-1}

5(a) State the Biot - Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^2}$$

$d\vec{B}$ = magnetic field (vector form) in weber / meter square

μ_0 = Permeability of free space $[4\pi \times 10^{-7}\ T\cdot m/A$

I = Current (Ampere)

$\vec{r}$ = unit vector

r^2 = distance between $d\vec{l}$ to P

$d\vec{l}$ = Point the direction of the current.

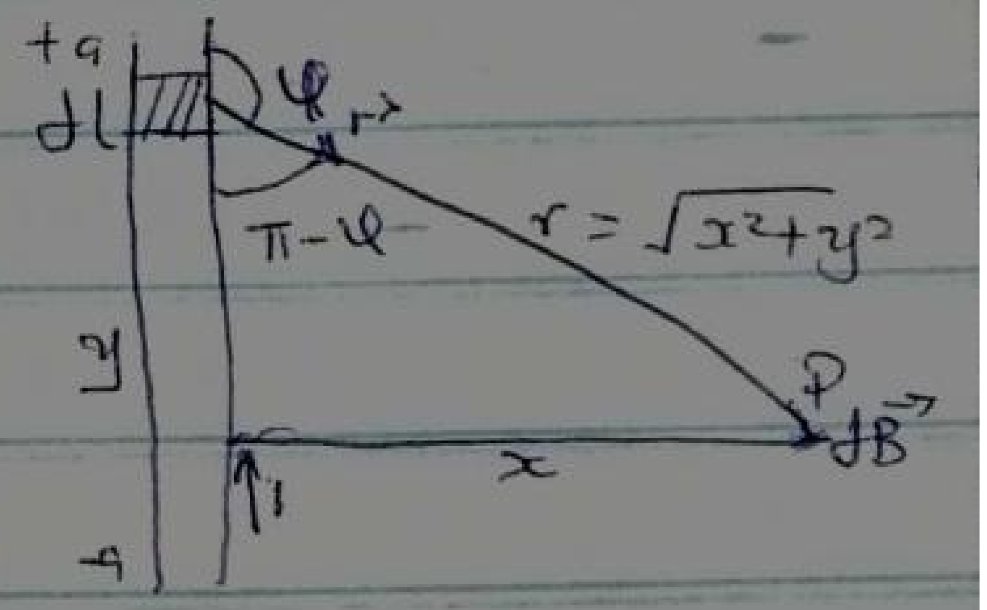
(b) Using the Biot - Savart Law, show that the magnitude ~~magnetic~~ of the magnetic field of a straight carrying conductor is given as

$$B = \frac{\mu_0 I}{2\pi r}$$

Solution

The magnitude of magnetic field is given as

$$B = \frac{\mu_0}{4\pi} \int_{-a}^{a} \frac{I\, d\vec{l} \sin\theta}{r^2}$$



$$\sin\theta = \sin(\pi - \varphi)$$

$$\therefore\ B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{d\vec{l} \sin(\pi - \varphi)}{r^2}$$

from diagram $r^2 = x^2 + y^2$ — *

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{—— 2}$$

But $\sin(\pi - \varphi) = \dfrac{x}{\sqrt{x^2 + y^2}} = \dfrac{x}{(x^2+y^2)^{1/2}}$ ---- * *

inputing * * int 2

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl\ x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}}\, dy \quad \text{——} ***$$

using Special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation *** becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

$$(x^2+a^2)^{1/2} \cong a \qquad a >>> x$$

$$a \longrightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{\cancel{4}\pi x} \cdot \frac{\cancel{2}\cancel{a}}{\cancel{a}}$$

$$B = \frac{\mu_0 I}{2\pi x}$$

$$x = r$$

$$B = \frac{\mu_0 I}{2\pi r}$$