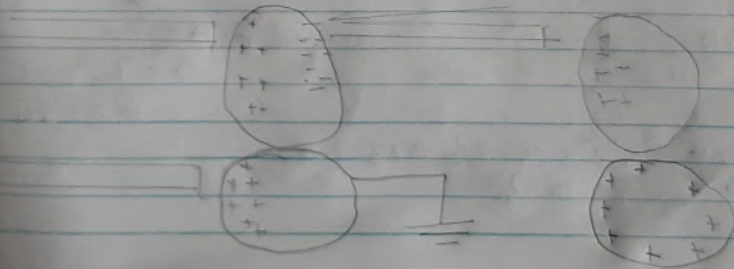


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PHY 102

Section A

1) Charging by induction

Electric Charges can be obtained on an object without touching it by a process called electrostatic induction. Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod & those in the sphere causes a redistribution of charges on the sphere furthest away from the rod. The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere as in (Fig 1.31b) some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere the induced positive charge remains on the ungrounded sphere & becomes uniformly distributed over the surface of the sphere.



$$1b) k = 9 \times 10^9$$

$$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$$

$$F = 1 \text{ N}$$

$$d = 2 \text{ m}$$

Charge on each sphere = ?

$$F = \frac{k q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1 q_2 \times 10^{-5})}{2^2}$$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2^2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2^2$$

quadratic equation

$$9 \times 10^9 q_2^2 - 4.5 \times 10^5 q_1 + 4 = 0$$

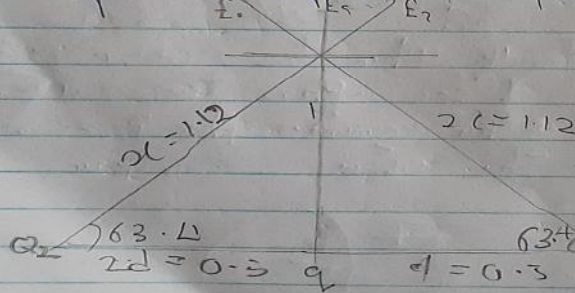
$$q_1 = 0.000011 \text{ C} \approx 1.1 \times 10^{-5} \text{ C}$$

$$q_2 = 0.000038 \text{ C} \approx 3.8 \times 10^{-5} \text{ C}$$

$$c) Q_1 = Q_2 = 8 \mu\text{C}$$

$$d = 0.5 \text{ m}$$

Q1 if electric field at a point P is zero



$$x^2 = 1^2 + 0.5^2$$

$$\sqrt{x^2} = \sqrt{1.25}$$

$$x = \sqrt{1.25}$$

$$x = 1.12$$

$$\tan \theta = \frac{\text{OPP}}{\text{ADJ}}$$

$$\tan \theta = \frac{1}{0.5}$$

$$\theta = \tan^{-1}(2)$$

$$\theta = 63.4$$

$$F = \frac{k q_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 57397.9598$$

$$\vec{E}_1 = kq_1 \frac{\vec{r}_1}{r_1^2} = 9 \times 10^9 \times 8 \times 10^{-6} \frac{1}{(1.2)^2} = 57397.95918$$

$$\vec{E}_2 = kq_2 \frac{\vec{r}_2}{r_2^2} = 9 \times 10^9 \times 2 = 1.8 \times 10^{10}$$

Vector	Angle	X-Component	Y-Component
$\vec{E}_1 = 57397.95918$	63.4°	$E_1 \cos \theta = 2570.046785$	$E_1 \sin \theta = 5132.262839$
$\vec{E}_2 = 57397.95918$	63.4°	2570.045785	5132.262839
$\vec{E}_3 = 9 \times 10^9 q$	90°	$E_3 \cos \theta = 0$ $E_x = 0$	$9 \times 10^9 q$ $E_y = 1026.52568$

$$\text{Magnitude} = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E_y = \sqrt{(0)^2 + (0.52568)^2}$$

$$\sin E_y = 0$$

$$\theta = 9 + 10^9 q + 1026.52568$$

Making q subject of formula

$$q = \frac{1026.52568}{9 \times 10^9}$$

$$q = 1.140582853 \times 10^{-16}$$

$$q = 11.411 \text{ e}$$

3a) Volume Charge Density $= \frac{dq}{dV} \quad n \, dV = \rho \, dV$

Surface Charge Density $\sigma = \frac{dq}{dA} \quad n \, dA = \sigma \, dA$

Linear Charge Density $= \frac{dq}{dL} \quad n \, dL = \lambda \, dL$

Electric Potential Difference

The Electric Potential Difference between two points in an electric field can be defined as the Work done per unit Charge against against electrical force —

When a charge is charged is transported from one point to another, it is measured in Volt (V) or Joules per Coulomb

(b/c) It is a scalar quantity

Elemental Work done dW is given by

$$dW = F \cdot dl \quad \text{--- (1)}$$

$$\text{But } F = -q_0 E \quad \text{--- (2)}$$

$$\text{Substituting equation (2) in (1) } dW = -q_0 E \cdot dl \quad \text{--- (3)}$$

Total work done in moving the test charge from A to B is

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E \cdot dl \quad \text{--- (4)}$$

From the definition of electric potential difference it follows that

$$V_B - V_A = \int_A^B (A \rightarrow B)_{q_0} \quad \text{--- (5)}$$

Putting eqn (4) in (5) yields $V_B - V_A = - \int_A^B E \cdot dl \quad \text{--- (6)}$

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Section B

a) Magnetic flux is defined as the strength of the magnetic field which can be represented by lines of force. It is represented by the symbol Φ . Mathematically given as $\Phi = B \cdot A$

$$\Rightarrow m = 9 \times 10^{-31} \text{ kg}$$

$$r = 1.1 \times 10^{-10} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ Weber/meter}^2$$

Cyclotron frequency = angular speed

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6.22 \times 10^{10} \text{ s}^{-1}$$

AC mass of electron, $= 9.1 \times 10^{-31}$ kg
radius $= 1.4 \times 10^{-7}$ m

magnetic field $= 3.5 \times 10^{-1}$ Weber / meter

Cyclotron frequency can be called angular speed

Recall that angular speed $\omega = \frac{v}{r} = \frac{qB}{m}$

$$\text{Substituting we have } \omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.1 \times 10^{-31}} \\ = 6.22 \times 10^{10} \text{ s}^{-1}$$

So cyclotron frequency $= 6.22 \times 10^{10} \text{ s}^{-1}$, the unit is equal to unit of frequency dimensionally

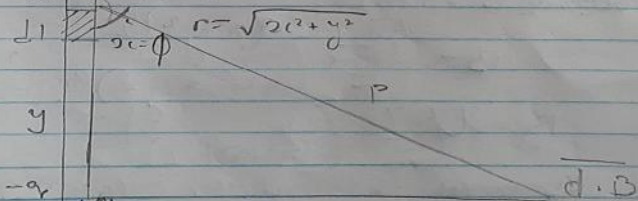
5.) Biot-Savart law states that the magnetic field is directly proportional to the product of permeability of free space (μ_0) the current (I), the change in length, the cross-sectional area proportional to the square of radius (r^2). It can be represented mathematically by

$$dB = \frac{\mu_0 I dl \sin \theta}{4\pi r^2} \quad \text{where } \mu_0 \text{ is a constant called permeability of free space}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{Wb}}{\text{A} \cdot \text{m}}$$

Unit of B is Weber / meter square

5.) Magnetic field of a straight current carrying conductor



A section of a straight current carrying conductor
Applying the Biot-Savart law, we find the magnitude of the field

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \phi}{r^2}$$

$$\sin(\alpha - \phi) = \sin \phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\alpha - \phi)}{r^2}$$

from diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\alpha - \phi)}{x^2 + y^2} \quad \text{--- (1)}$$

$$\text{But } \sin(\alpha - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{--- (2)}$$

Substituting (2) into (1), $B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \cdot x}{(x^2 + y^2)^{3/2}}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{dy}{(x^2 + y^2)^{3/2}}$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \text{--- (3)}$$

Using special integrals: $\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \cdot \frac{y}{(x^2 + y^2)^{1/2}}$

Equation (3) becomes $B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I x}{2\pi} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very much longer than compared to its distance x from point, we consider it infinitely long. That B , when a is much larger than x , $(x^2 + a^2)^{1/2} \approx a$

then $\frac{1}{\sqrt{a^2 + z^2}} \approx \frac{1}{a} \left(1 - \frac{z^2}{2a^2} \right)$

$$\vec{B} = \frac{\mu_0 I}{2\pi a}$$

In a physics situation, we have axial symmetry about the y -axis. Thus at all points in a circle of radius r around the conductor the magnitude of B is $\frac{\mu_0 I}{2\pi r}$.

Magnitude of the magnetic field of flux density B near a long straight current carrying conductor.