

QUESTION 2

Keyboard $\xrightarrow{\text{Produced}}$ Kansas & Mexico $\xrightarrow{\text{consumed}}$ New York & California

Kansas \rightarrow 15 cartons of keyboards } production
 Mexico \rightarrow 8 cartons

New York \rightarrow 10 } consumption
 California \rightarrow 13

\$4 cost Mexico - New York - x_1
 \$1 " Mexico - California - x_2
 \$2 " Kansas - New York - x_3
 \$3 " Kansas - California - x_4

minimize the overall transportation cost

$$W = 4x_1 + x_2 + 2x_3 + 3x_4$$

subject to

$$x_1 + x_3 \leq 10$$

$$x_2 + x_4 \leq 13$$

$$x_1 + x_2 \leq 8$$

$$x_3 + x_4 \leq 15$$

where x_1, x_2, x_3, x_4

} constraints

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 10 \\ 0 & 1 & 0 & 1 & 13 \\ 1 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 1 & 15 \\ \hline 4 & 1 & 2 & 3 & 0 \end{array} \right]$$

Transposing

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ \hline 10 & 13 & 8 & 15 & 0 \end{array} \right]$$

finding max value of $w = 10y_1 + 13y_2 + 8y_3 + 15y_4$
subject to constraints

four slack variables s_1, s_2, s_3, s_4

$$\left. \begin{array}{l} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \right\} \begin{array}{l} y_1 + y_3 \leq 4 \\ y_2 + y_3 \leq 1 \\ y_1 + y_4 \leq 2 \\ y_2 + y_4 \leq 3 \end{array}$$

where $0 = -10y_1 + 13y_2 - 8y_3 - 15y_4$

y_1	y_2	y_3	y_4	s_1	s_2	s_3	s_4	
1	0	1	0	1	0	0	0	4
0	1	1	0	0	1	0	0	1
1	0	0	1	0	0	1	0	2
0	1	0	1	0	0	0	1	3
-10	-13	-8	-15	0	0	0	0	0

$$\left. \begin{array}{l} 4 \div 0 \rightarrow 0 \\ 1 \div 0 \rightarrow 0 \\ 2 \div 1 \rightarrow 2 \\ 3 \div 1 \rightarrow 3 \end{array} \right\}$$

↑

To get new row 4

$\rightarrow -1R_3 + R_4 \rightarrow R_4$

$$\left[\begin{array}{cccccccc|c} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 0 & 0 & 0 & -1 & 1 & 1 \\ -10 & -13 & -4 & -15 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

To get new row 5 $+15R_3 + R_5 \rightarrow R_5$

$$\left[\begin{array}{cccccccc|c} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 0 & 0 & 0 & -1 & 1 & 1 \\ \hline -5 & -13 & -8 & 0 & 0 & 0 & 15 & 0 & 30 \end{array} \right] \begin{array}{l} 4 \div 0 = x \\ 1 \div 1 = 1 \\ 2 \div 0 = x \\ 1 \div 1 = 1 \end{array}$$

\uparrow To get new row 4

$-1R_2 + R_4$

$$\left[\begin{array}{cccccccc|c} y_1 & y_2 & y_3 & y_4 & s_1 & s_2 & s_3 & s_4 & \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ -1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ \hline -5 & 0 & 5 & 0 & 0 & 13 & 15 & 0 & 43 \end{array} \right]$$

To get new row 5 $+13R_2 + R_5 \rightarrow R_5$

$y_2 = 1$
 $y_4 = 2$
 $s_1 = 4$

from the simplex table, maximum value of

$w = 43$

original problem (minimization)

$w = 43$

when $s_1 = 4$, $s_2 = 13$, $s_3 = 15$, $s_4 = 0$

QUESTION 1

First step: Definition of variables

w_i = number of workers during i th month

$$w_0 = 30$$

x_i = carpets made per month \rightarrow \$2000 salary

y_i = number of carpets made in the i th month

a_i, b_i = number of workers hired / fired respectively

s_i = no^o carpets stored; $s_0 = 0$ (no initial surplus for storage)

constraints

$$w_0, w_i, x_i, y_i, a_i, b_i, s_i, s_0 \geq 0, (i = 1, \dots, 12)$$

Total number of carpets

$$y_i = 20w_i + x_i$$

no workers at the start of the month

$$w_i = w_{i-1} + (a_i - b_i)$$

stored carpet

$$s_i = s_{i-1} + y_i - d$$

where $d \rightarrow$ carpet demand per month

Hiring costs \$320, firing cost \$400 per worker

minimization of cost

$$= 2000 \sum w_i + 320 \sum a_i + 400 \sum b_i + 8 \sum s_i + \sum y_i + 480 \sum x_i$$