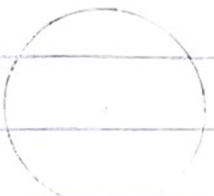
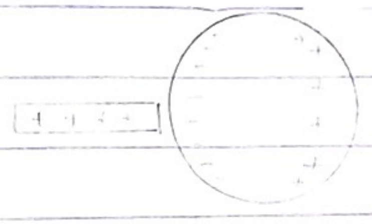
1.

a. A positively charged rod is brought near a neutral sphere which is insulated so there is no connection to ground. The electrons in the sphere migrate towards the side closest to the positively charged rod, so the region farthest from the rod has an excess of positive charge, the sphere is polarized.

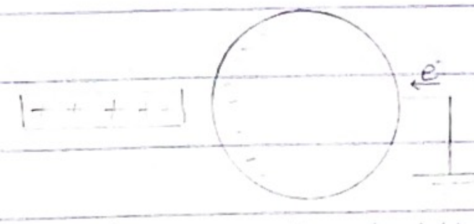
When a grounded conducting wire is connected to the sphere, electrons enter the sphere, attracted to the +vely charged rod, and the +ve charges in the sphere. The wire is removed, leaving the sphere with an excess of negative charge. Finally, the rod is removed, the induced negative charge becoming uniformly distributed over the sphere.



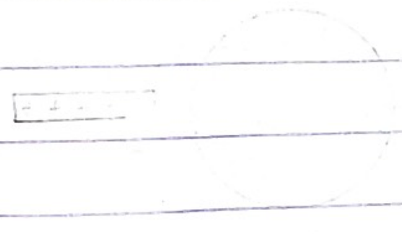
ai) [illegible] sphere



aii) Sphere becomes polarized



aiii) Electrons enter sphere, attracted to the +charge



aiv) Excess of -ve charge

av) Electrons redistribute normally

b. Let the spheres be q_1 & q_2

$F_{12} = 1.0N$, $q_1 + q_2 = 5.0 \times 10^{-5} C$

$r = 2.0m$, $r^2 = 4m^2$

From Coulomb's Law; $F = \frac{kq_1q_2}{r^2}$

$$\frac{Fr^2}{k} = q_1q_2 = \frac{1 \times 4}{9 \times 10^9} = 4.44 \times 10^{-10} C^2$$

$q_1 + q_2 = 5.0 \times 10^{-5} C$ — ①

$q_1 q_2 = 4.44 \times 10^{-10} C^2$ — ②

$q_1 = 5.0 \times 10^{-5} C - q_2$ — ③

$\therefore q_2 (5.0 \times 10^{-5} C - q_2) = 4.44 \times 10^{-10} C^2$

$5.0 \times 10^{-5} q_2 - (q_2)^2 = 4.44 \times 10^{-10}$

$(q_2)^2 - 5.0 \times 10^{-5} q_2 + 4.44 \times 10^{-10} = 0$

Using the quadratic formula;

$q_2 = 3.86 \times 10^{-5} C$ or $1.14 \times 10^{-5} C$

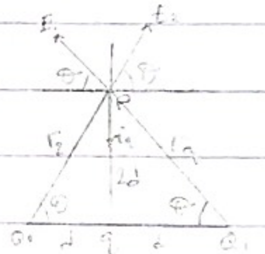
from (3);

$q_1 = 5.0 \times 10^{-5} C - q_2$

$\therefore q_1 = 1.14 \times 10^{-5} C$ or $3.86 \times 10^{-5} C$

$q_2 = 3.86 \times 10^{-5} C$ or $1.14 \times 10^{-5} C$

6\. For the electric field at P to be zero;



$\theta = \tan^{-1}\left[\frac{1}{0.5}\right] = \tan^{-1}(2)$

$\theta = 63.4^\circ$

$\vec{E}_{net} = 0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_q$

$E_1 = \frac{kq_1}{r_1^2} = \frac{(9 \times 10^9) \times (8 \times 10^{-6})}{(1.118)^2}$

[Pythagoras theorem, $r_q^2 = (2d)^2 + d^2$, $r_q = \sqrt{5d^2}$
$d = 0.5m$, $r_q = \sqrt{1.25} = 1.118\,m$]

$E_1 = 5.76 \times 10^4 N/C$

$E_2 = 5.76 \times 10^4 N/C$

$E_q = \frac{(9 \times 10^9) \times q}{(1)^2} = 9 \times 10^9 q \; N/C$

E	θ	x	y
E_q	90°	$9 \times 10^9 q \cos 90° = 0$	$+9 \times 10^9 q \sin\theta = +kq$
E_1	63.4°	$-5.76 \times 10^4 \cos 63.4$	$+5.76 \times 10^4 \sin 63.4$
E_2	63.4°	$+5.76 \times 10^4 \cos 63.4$	$+5.76 \times 10^4 \sin 63.4$

$\Sigma E_x = 0$ $\quad \Sigma E_y = 1.03 \times 10^5 + kq$

$\vec{E}_{net} = 0 = \sqrt{0^2 + (1.03 \times 10^5 + kq)^2}$

$0^2 = \sqrt{0 + (1.03 \times 10^5 + kq)^2}$ $\quad \therefore 0 = 1.03 \times 10^5 + kq$

~~Taking square root~~

$kq = -1.03 \times 10^5$

$q = \dfrac{-1.03 \times 10^5}{9 \times 10^9} = -11 \times 10^{-6} C = -11 \mu C$

2.

a. An Electric Field is a region in space where an electric charge will experience an electric force, while Electric Field Intensity is the force per unit charge acting on a charged particle in an electric field.

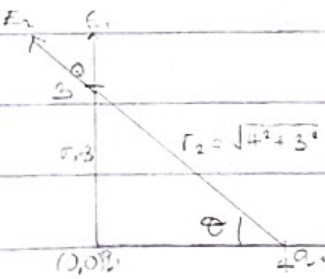
b. i. q_1 at 0, q_2 at 4, $E_{net} = ?$ at 7 (P)

$q_1 = 8 \times 10^{-9} C$, $q_2 = 12 \times 10^{-9} C$, $r_1 = 7m$, $r_2 = 3m$

$E_1 = \dfrac{kq_1}{r_1^2} = \dfrac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.47 N/C$

$E_2 = \dfrac{kq_2}{r_2^2} = \dfrac{(9 \times 10^9)(12 \times 10^{-9})}{3^2} = 12 N/C$

$E_{net} = E_1 + E_2 = 13.47 N/C \simeq 13.5 N/C$



ii. $E_1 = \dfrac{kq_1}{r_1^2} = \dfrac{(9 \times 10^9)(8 \times 10^{-9})}{3^2} = 8 N/C$

$r_2 = \sqrt{4^2 + 3^2} = 5m$

$E_2 = \dfrac{kq_2}{r_2^2} = \dfrac{(9 \times 10^9)(12 \times 10^{-9})}{5^2} = 4.32 N/C$

$\tan\theta = \dfrac{3}{4}$ $\theta = \tan^{-1}(0.75)$

$\theta = 37°$

E	θ	x	y
E_1	90°	$+8\cos 90 = 0$	$+8\sin 90 = 8$
E_2	37°	$-4.32\cos 37 = -3.45$	$+4.32\sin 37 = 2.59$

$\Sigma E_x = -3.45$ $\Sigma E_y = 10.59$

$E_{net} = \sqrt{(-3.45)^2 + (10.59)^2} = \sqrt{124} = 11.14 N/C$

5.

a. The Biot-Savart Law asserts that, the magnetic intensity at any point due to a steady current in an infinitely long wire is directly proportional to the current, and inversely proportional to the square of the distance from each point to the wire.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$ μ_0 - permeability of free space

r - distance from dl to P

The magnitude of the total magnetic field is,

$$B = \frac{\mu_0}{4\pi}\int \frac{I\,dl \sin\theta}{r^2}.$$

b. 

A section of a straight current carrying conductor

Applying the Biot-Savart law, the magnitude of field $d\vec{B}$ is,

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin\theta}{r^2} \qquad B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{r^2} \quad \left[\sin\theta = \sin(\pi-\varphi)\right.$$

From the diagram, $r^2 = x^2+y^2$ [Pythagoras theorem]

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a}\frac{dl\sin(\pi-\varphi)}{x^2+y^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}} = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl\,\frac{x}{(x^2+y^2)^{3/2}} \quad \left|\ \sin\theta = \frac{x}{\sqrt{x^2+y^2}} = \sin(\pi-\varphi)\right.$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a}\frac{1}{(x^2+y^2)^{3/2}}\,dy \quad [dy = dl]$$

Using special integrals;

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a} = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When length $2a$ of the conductor is >> than distance x from point P, it is considered infinitely long $(a >> x)$. $\therefore (x^2+a^2)^{1/2} \cong a$ as $a \to \infty$ $\therefore B = \frac{\mu_0 I}{2\pi x}$. At all points in circle of radius r around the conductor; $B = \frac{\mu_0 I}{2\pi r}$

6.

a. In an electric guitar, the coil [the pickup coil] is placed near the vibrating guitar string which is made up of a metal that can be magnetised. A permanent magnet inside the coil magnetizes the portion on the string nearest the coil. When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil. The changing flux induces an emf in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeaker, which produces the sound waves we hear.

b. $N = 300$ $\quad A = (0.1)^2 = 0.01 m^2$ $\quad |e| = ?$

$R = 2.0\Omega$ $\quad \Delta\phi_B = 10T$ $\quad \Delta t = 0.5s$

i. Induced emf $|e| = \frac{N\Delta\phi}{\Delta t} = \frac{300 \times 0.01 \times 10}{0.5} = 60V$

ii. Induced current $= \frac{|e|}{R} = \frac{60}{2} = 30A$

c. $A = 0.05 \times 0.08 = 4 \times 10^{-3} m^2$ $\quad I = 0.1A$

$N = 75$ $\quad R = 8\Omega$ $\quad \Delta\phi_B = ?$ $\quad \Delta t = ?$

$I = \frac{|e|}{R}$ $\quad |e| = 8 \times 0.1A = 0.8V$

$|e| = N\frac{\Delta B}{\Delta t}$ $\quad \therefore \frac{\Delta B}{\Delta t} = \frac{0.8}{75 \times 4 \times 10^{-3}} = 2.7 T/s$