

NAME: OFODI CHRISTABEL

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1) If $A = 5i - 7j - 6k$, $B = j + 4k$, $C = 9i - 4j + k$, Find $-8(A+B) \cdot (C-A)$

Solution

$$(A+B) = (5i - 7j - 6k) + (j + 4k)$$

$$= 5i - 6j - 2k$$

$$-8(A+B) = -8(5i - 6j - 2k)$$

$$= -40i + 48j + 16k$$

$$(C-A) = (9i - 4j + k) - (5i - 7j - 6k)$$

$$= 4i + 3j + 7k$$

$$-8(A+B) \cdot (C-A) = (-40i + 48j + 16k) \cdot (4i + 3j + 7k)$$

$$= -160i + 144j + 112k$$

2) Find the unit vector tangent to the space curve $x = 3t$, $y = t^2$ and $z = 4t^3$ at the point where $t = 1$

Solution

$$r = 3ti + t^2j + 4t^3k$$

$$\frac{dr}{dt} = 3i + 2tj + 12t^2k$$

$$\text{at } t = 1, \frac{dr}{dt} = 3i + 2j + 12k$$

$$\left| \frac{dr}{dt} \right| = \sqrt{3^2 + 2^2 + 12^2} = \sqrt{9 + 4 + 144} = \sqrt{157} = 12.3$$

$$\text{Hence, } \frac{3i + 2j + 12k}{12.3}$$

3. A particle moves along a curve, $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$. Where t is time, Find its acceleration

Solution

$$\text{Acceleration} = \frac{d^2r}{dt^2}$$

To find the acceleration we must first find $\frac{dr}{dt}$

$$r = xi + yj + zk$$

$$r = (8t^2)i + (t^2 - 4t)j + (t + 1)k$$

$$\frac{dr}{dt} = (16t)i + (2t - 4)j + k$$

$$\frac{d^2r}{dt^2} = 16i + 2j$$

4. If $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$. Find $(\bar{A} \times \bar{B}) \times \bar{C}$

Solution

$\bar{A} \times \bar{B}$	i	j	k
	1	2	-4
	2	-3	1

$$i \begin{vmatrix} 2 & -4 \\ -3 & 1 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 2 & 1 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix}$$

$$i(2 - 12) - j(1 - (-8)) + k(-3 - 4)$$

$$i(-10) - j(9) + k(-7) = -10i - 9j - 7k$$

$(\bar{A} \times \bar{B}) \times \bar{C}$	i	j	k
	-10	-9	-7
	0	4	-3

$$i \begin{vmatrix} -9 & -7 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -10 & -7 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} -10 & -9 \\ 0 & 4 \end{vmatrix}$$

$$i(27 - (-24)) - j(30 - 0) + k(-40 - 0)$$

$$i(51) - j(30) + k(-40) = 51i - 30j - 40k$$

5) Given $R = 4\sin 3t \mathbf{i} + 4e^{3t} \mathbf{j} + 7t^3 \mathbf{k}$, find the integral of R with respect to t from 0 to 1

Solution

$$\int_0^1 R = \int_0^1 (4\sin 3t + 4e^{3t} + 7t^3) dt$$

$$= \int_0^1 4\sin 3t dt + \int_0^1 4e^{3t} dt + \int_0^1 7t^3 dt$$

$$= \left. \frac{-4\cos(3t)}{3} \right|_0^1 + \left. \frac{4e^{3t}}{3} \right|_0^1 + \left. \frac{7t^4}{4} \right|_0^1$$

$$\left(\frac{-4\cos 3(1)}{3} \right) - \left(\frac{-4\cos 3(0)}{3} \right) + \mathbf{j} \left(\frac{4e^{3(1)}}{3} \right) - \left(\frac{4e^{3(0)}}{3} \right) + \mathbf{k} \left(\frac{7(1)^4}{4} \right) - \left(\frac{7(0)^4}{4} \right)$$

$$= \mathbf{i} \left(\frac{-4\cos 3 + 0}{3} \right) + \mathbf{j} \left(\frac{4e^3}{3} - \frac{4}{3} \right) + \mathbf{k} \left(\frac{7}{4} \right)$$

$$= \frac{-4\cos 3}{3} \mathbf{i} + \frac{4e^3 - 4}{3} \mathbf{j} + \frac{7}{4} \mathbf{k}$$

