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MATRIC NUMBER: 19/sci01/070

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1. Find the derivative of the following using first principle

a. $y = \sin(3/x^2)$

b. $y = 4/x^3$

Solution

a. $y = \sin(3/x^2)$

b. $\cancel{\Delta y} = \dfrac{dy}{dx} = \dfrac{-6\cos\left(\frac{3}{x^2}\right)}{x^3}$

b. $y = 4/x^3$

$$y + \Delta y = \frac{4}{(x+\Delta x)^3}$$

$$\Delta y = \frac{4}{(x+\Delta x)^3} - \frac{4}{x^3}$$

$$\Delta y = \frac{4x^3 - 4(x+\Delta x)^3}{x^3(x+\Delta x)^3}$$

Divide through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{\dfrac{4x^3 - 4x^3 - 12x^2\Delta x + 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3)}}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{\Delta x}(-12x^2\Delta x + 12x(\Delta x)^2 - 4(\Delta x)^3)}{x^3(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3)} \times \frac{1}{\Delta x}$$

$$= \frac{-12x^2\Delta x + 12x(\Delta x)^2 - 4(\Delta x)^3}{x^3(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3)}$$

$$\lim_{\Delta x \to 0} = \frac{-12x^2}{x^3(x^3)} = \frac{-12\cancel{x^2}}{x^{\cancel{6}\,4}} = \frac{-12}{x^4}$$

2. Find the Integral of

a. $dx/(x^2+36)$

b. $dx/(x^2+13)$

Solution

a. $$\int \frac{dx}{x^2+36}\,dx = \int \frac{1}{36\left(1+\frac{x^2}{36}\right)}\,dx = \frac{1}{36}\int \frac{1}{1+\left(\frac{x}{6}\right)^2}\,dx$$

Let $u = x/6$

$du = 1/6\,dx$

$dx = 6\,du$

$$= \frac{1}{36}\int \frac{1}{1+u^2}\,6\,du = \frac{6}{36}\int \frac{1}{1+u^2}\,du = \frac{1}{6}\int \frac{1}{1+u^2}\,du$$

Remember $\int \frac{1}{1+u^2}\,du = \arctan u$

$\therefore \quad \frac{1}{6}\int \frac{1}{1+u^2}\, du = \frac{\arctan u}{6}$

when $u = \frac{x}{6}$

$= \frac{\arctan(\frac{x}{6})}{6}$

b) $\int \frac{dx}{(x^2+13)}\, dx = \int \frac{1}{x^2+13}$

Let $u = \frac{x}{\sqrt{13}}$

$\frac{du}{dx} = \frac{1}{\sqrt{13}}$

$dx = \sqrt{13}\, du$

$= \int \frac{\sqrt{13}}{13u^2+13} \cdot du$

$= \frac{1}{\sqrt{13}}\int \frac{1}{u^2+1}\, du$

Remember :

$\int \frac{1}{u^2+1}\, du = \arctan u$

$\therefore \frac{1}{\sqrt{13}}\int \frac{1}{u^2+1}\, du$

$= \frac{\arctan u}{\sqrt{13}}$

when $u = \frac{x}{\sqrt{13}}$

$= \frac{\arctan(\frac{x}{\sqrt{13}})}{\sqrt{13}}$ //