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15/ENG02/012

Question 1

Assigning and defining variables:

a_i = no. of workers during i^{th} month;
x_i = no. of carpets made during i^{th} month.
y_i = no of carpets made by overtime in month i.
h_i = no. of workers hired at beginning of month i;
f_i = no. of workers fired at beginning of month i;
z_i = no. of carpets stored at the end of month i.

$w_0 = 30$

$z_0 = 0$

There are 72 variables in total for 12 months

$a_i, x_i, y_i, h_i, f_i, z_i \geq 0$ &

where $i = 1, 2, \ldots, 12$

~~74 variables if we include~~.

Total no. of carpets made per month;

$$x_i = 20a_i + y_i$$

for each value of i the is one constraint

The no. of worker can change at the start of each month depending of workers hired and worker fired

$$! - \quad a_i = a_{i-1} + h_i - f_i$$

The no. of carpets stored at end of each month, is what is started with at the beginning of the ~~end~~ month, plus ~~the~~ no. of carpets made, minus demand for the month

$z_i = z_{i-1} + x_i - d_i$

where d_i – demand of carpets for the next

and the overtime is limited.

$y_i \le 6a_i$

The objective function is to minimize the total cost;

Therefore objective function is:

$$\min \; 2000\sum_i a_i + 320\sum_i h_i + 400\sum_i f_i + 8\sum_i z_i + 180\sum_i y_i$$

$$\min \; 2000\sum_i a_i + 320\sum_i h_i + 400\sum_i f_i + 8\sum_i z_i + 180\sum_i y_i$$

Classwork 2

Question 2

Produced

Kansas — 15 cartons

Mexico — 8 "

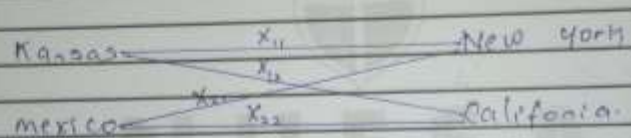
Consumed

New York — 10 cartons

California — 13 "

Transportation Cost (\$)

	New York	California
Kansas	2	3
mexico	4	1



$$\left.\begin{array}{l} X_{11} + X_{12} \ge 15 \\ X_{21} + X_{22} \ge 8 \end{array}\right\} \text{Produced Constraints}$$

$$\left.\begin{array}{l} X_{11} + X_{21} \ge 10 \\ X_{12} + X_{22} \ge 13 \end{array}\right\} \text{Consumed Constraints}$$

The Objective function is;

$a_{11}X_{11} + a_{12}X_{12} + a_{21}X_{21} + a_{22}X_{22}$

$= 2X_{11} + 3X_{12} + 4X_{21} + X_{22}$

subject to;

$$x_{11} + x_{12} \geq 15$$
$$x_{21} + x_{22} \geq 8$$
$$x_{11} + x_{21} \geq 10$$
$$x_{12} + x_{22} \geq 13$$
$$\forall\, x_{ij} \geq 0 \; ; \; \begin{matrix} i = 1,2 \\ j = 1,2 \end{matrix}$$

To convert the problem to a maximization problem, first form the augmented matrix for this system of inequalities.

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 15 \\ 0 & 0 & 1 & 1 & 8 \\ 1 & 0 & 1 & 0 & 10 \\ 0 & 1 & 0 & 1 & 13 \\ \hline 2 & 3 & 4 & 1 & 0 \end{array}\right]$$

Next find the transpose of this matrix.

$$\begin{array}{ccccc} x_{11} & x_{12} & x_{21} & x_{22} & \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 & 1 \\ 15 & 8 & 10 & 13 & 0 \end{array}$$

This applies that the dual max. problem is:

$$\underbrace{z = 15x_{11} + 8x_{12} + 10x_{21} + 13x_{22}}_{\text{Dual objective function}}$$

$X_{11} + X_{21} \leq 2$
$X_{11} + X_{22} \leq 3$
$X_{12} + X_{21} \leq 4$
$X_{12} + X_{22} \leq 1$

} Dual Constraints

where $X_{11} \geq 0$, $X_{12} \geq 0$, $X_{21} \geq 0$, $X_{22} \geq 0$
we now apply simplex method to the dual problem

X_{11}	X_{12}	X_{21}	X_{22}	S_1	S_2	S_3	S_4	Z	
1	0	1	0	1	0	0	0	2	S_1
1	0	0	1	0	1	0	0	3	S_2
1	0	0	0	0	0	1	0	4	S_3
0	1	1	0	0	0	0	1	1	S_4
0	1	0	1	0	0	0	1	1	S_4
$\boxed{-15}$	-8	-10	-13	0	0	0	0	0	

after introducing the slack variables
where $X_{11}, X_{12}, X_{21}, X_{22}, S_1, S_2, S_3, S_4 \geq 0$
-15 is the most negative number

$R_1 = \frac{1}{1} R_1$

1	0	1	0	1	0	0	0	2	X_{11}
1	0	0	1	0	1	0	0	3	S_2
0	1	1	0	0	0	1	0	4	S_3
0	1	0	1	0	0	0	1	1	S_4
-15	-8	-10	-13	0	0	0	0	0	

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3$
$R_4 \rightarrow R_4$
$R_5 \rightarrow R_5 + 15R_1$

1	0	1	0	1	0	0	0	2
0	0	-1	+1	-1	+1	0	0	+1
0	1	1	0	0	0	1	0	4
0	1	0	1	0	0	0	1	1
0	-8	5	-13	15	0	0	0	30

-13 is the most negative.

$R_1 \to R_1$
$R_2 \to R_3$
$R_2 \to R_4 + R_2$
$R_5 \to R_5 + 13R_2$

1	0	1	0	1	0	0	0	2
0	0	-1	1	-1	1	0	0	1
0	1	1	0	0	0	1	0	4
0	1	1	0	1	-1	0	1	0
0	-8	-8	0	2	13	0	0	43

$R_4 \to R_4 - R_3$
$R_5 \to R_5 + 8R_3$
$R_1 \to R_1$
$R_2 \to R_2$
$R_3 \to R_3$

X_{11}	X_{12}	X_{21}	X_{22}	S_1	S_2	S_3	S_4	Z
1	0	1	0	1	0	0	0	2
0	0	-1	1	-1	1	0	0	1
0	1	1	0	0	0	1	0	4
0	0	0	0	1	-1	-1	1	-4
0	0	0	0	2	13	8	0	75
				↑	↑	↑	↑	
				X_{11}	X_{12}	X_{21}	X_{22}	

Therefore the maximum value of Z
$Z = 75$

Which occurs when $X_{11} = 2$
$X_{12} = 13$
$X_{21} = 8$
$X_{22} = 0$