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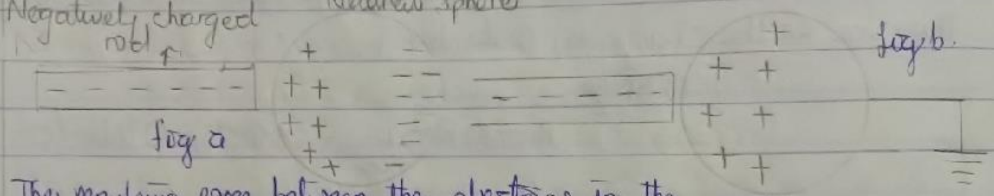
Name: Ogwudu Nkechi Martina.
Matric No: 19/sc001/072.
Department: Computer Science
Covid-19 Holiday Assignment.

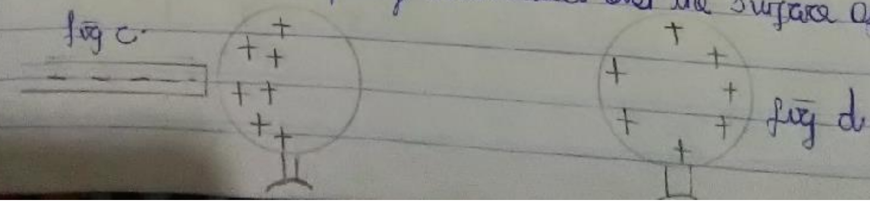
1a Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

Solution

To produce a negatively charged wire by method of induction, consider a negatively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conducting path to ground (i.e to no flow of electrons) as shown in the diagram below



The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farther away from the rod (fig a). The region of the sphere nearest the negatived charged has excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, as in fig b, some of the electrons leave the sphere and travel to the earth and when the wire to ground is removed (fig c) the sphere is left with excess induced positive charge. Finally, when the rubber rod is taken away from the region of the sphere (fig d) the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere



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1b. Solution.

Combined charge $q_1 + q_2 = 5.0 \times 10^{-5}C$

$F = 1.0N$, $r = 2.0m$, $K = 9 \times 10^9 Nm^2/C^2$

From Columbs law

$$F = \frac{Kq_1q_2}{r^2}$$

Let the product of the charges be $q_{(p)}$.

$$q_{(p)} = \frac{(1.0)N(2.0m)^2}{9 \times 10^9}$$

$$= 4.44 \times 10^{-10} C^2$$

$q_1 + q_2 = 5.0 \times 10^{-5}C$ — (1)

$q_1 q_2 = 4.449 \times 10^{-10} C^2$ — (2)

$q_1 = 5.0 \times 10^{-5} - q_2$ — (3)

Substitute $q_1 = 5.0 \times 10^{-5} - q_2$ into equation (2)

$= (5.0 \times 10^{-5} - q_2) \times q_2 = 4.449 \times 10^{-10}$

$= 5.0 \times 10^{-5} q_2 - q_2^2 = 4.449 \times 10^{-10}$

$= -q_2^2 + (5.0 \times 10^{-5}) q_2 = 4.449 \times 10^{-10}$

$= q_2^2 - (5.0 \times 10^{-5}) q_2 + 4.449 \times 10^{-10} = 0$

Using quadratic formula.

$$q_2 = \frac{(5.0 \times 10^{-5}) \pm \sqrt{(5 \times 10^{-5})^2 - 4(4.449 \times 10^{-10})}}{2}$$

$q_2 = 3.840 \times 10^{-5}$ since the charges are positive

Put $q_2 = 3.840 \times 10^{-5}$ into equation (1).

$q_1 = 5.0 \times 10^{-5} - 3.840 \times 10^{-5}$

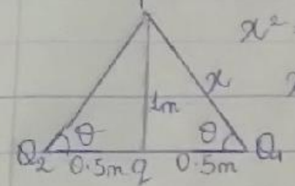
$q_1 = 1.157 \times 10^{-5}$

$\simeq 1.16 \times 10^{-5}$

$q_1 = 1.16 \times 10^{-5} C$

$q_2 = 3.84 \times 10^{-5} C$.

C Solution $Q_1 = Q_2 = 8\mu C$, $d = 0.5m$



$x^2 = 1^2 + 0.5^2$

$x = \sqrt{1^2 + 0.5^2}$

$x = 1.12m$

$E_1 = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2}$

$E_1 = 57397.959$

Since $Q_1 = Q_2$, $E_1 = E_2 = 57397.959$

$E_q = 9 \times 10^9 \times \frac{q}{1} = 9 \times 10^9 q$

To find θ = $\tan\theta = \frac{1}{0.5} = 2$

$\theta_1 = \tan^{-1}(2) = 63.43^\circ$

$\theta_1 = \theta_2 = 63.43^\circ$

$\theta_q = 90^\circ$

Vector	θ	x-component	y-component
E_1	63.4°	$E_1 \times \cos\theta =$ 25673.15	$E_1 \sin\theta =$ 51336.08
E_2	63.4°	$E_2 \times \cos\theta$ $= -25673.15$	51336.08
E_q	90°	$E_q \cos\theta =$ 0	$E_q \sin 90^\circ =$ $9 \times 10^9 q$
		$\Sigma x = 0$	$\Sigma y = \cancel{10126}$ 102672.16

E magnitude $= \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

$= \sqrt{(0)^2 + (102672.16)^2}$

$E_q = 102672.16$, $E = 0$.

$102672.16 - 9 \times 10^9 q = 0$.

$q = \frac{-102672.16}{9 \times 10^9} = -1.14 \times 10^{-5}$

$q = -11.4 \mu C$

3a.

(i) Volume charge density,

(ii) Surface charge density,

(iii) Linear charge density,

3b. ELECTRIC POTENTIAL DIFFERENCE

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt or Joules per Coulomb. Electric potential difference is a scalar quantity.

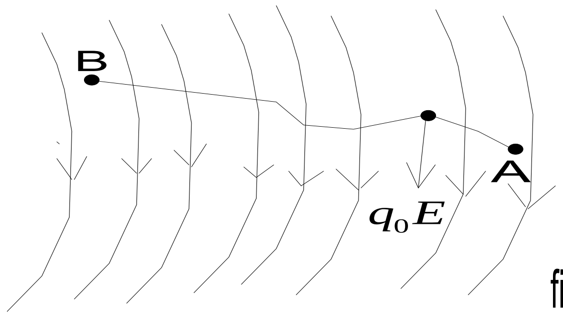


fig. 4.1

Consider the diagram above, suppose a test charge is moved from point B to point A along an arbitrary path inside an electric field. The electric field exerts a force on the charge as shown in fig 4.1. To move the test charge from to at constant velocity, an external force of must act on the charge. Therefore, the elemental work done is given as:

But

Substituting equation in yields

Then total work done in moving the test charge from to is:

From the definition of electric potential difference, it follows that:
Putting equation in yields

3c.

5b. Solution

$-2\mu C \quad 4-x \quad x \quad 10\mu C$

$Q_2 \quad 4m \quad Q_1$

$x_1 = 0, \quad x_2 = 4m$

Find v (potential difference) = 0.

$$V = \frac{kQ}{r}$$

Starting from the right side, let $r = x$

$$V_P = \frac{1}{4\pi\varepsilon_0} \times \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right)$$

$$V_P = 9\times10^9 \times \left[\frac{10\times10^{-6}}{x} + \frac{(-2\times10^{-6})}{4-x}\right]$$

where $V_P = 0$

$$9\times10^9 \times \left[\frac{10\times10^{-6}}{x} - \frac{2\times10^{-6}}{4-x}\right].$$

$$9\times10^9 \times \left[\frac{(10\times10^{-6})(4-x) - (2\times10^{-6})(x)}{(x)(4-x)}\right] = 0.$$

$$= 9\times10^9 \times \left[4\times10^{-5} - 10\times10^{-6}x - 2\times10^{-6}x\right] = 0.$$

$$9\times10^9 \times \left[\frac{4\times10^{-5} - 1.2\times10^{-5}x}{4x - x^2}\right] = 0.$$

$$= \frac{360000 - 108000x}{4x - x^2} = 0.$$

$$360000 - 108000x = 0.$$

$$x = \frac{360000}{108000}$$

$$x = 3.33m$$

SECTION B.

4a. magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol Φ.mathematically given as $\Phi = B.\, d\,A$

4b. Solution

mass, $m = 9.11 \times 10^{-31}$ kg

radius, $r = 1.4 \times 10^{-7}$ m

magnetic field, $B = 3.5 \times 10^{-1}$ weber/m²

Cyclotron Frequency = angular speed.

$$\omega = \frac{v}{r} = \frac{qB}{m}.$$

$$\therefore \quad \omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6.222 \times 10^{10}\, T^{-1}$$

4c. In the question we were given paramiters such as

i.mass of the electron $=9.11 \times 10^{-31}$ kg

ii.A radius of 1.4×10^{-7}m

iii.magnetic field of 3.5×10^{-1}weber\meter square

and you are asked to find the cyclotron frequency which is equal or the same thing as angular speed.it is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall that angular speed is given as ω=qB/m

Substituting we have $\omega = \dfrac{1.6x10^{-10} x 3.5x10^{-10}}{9.11x10^{-31}}$

$=6.22\times10^{10}T^{-1}$

So since cyclotron frequency is equal to angular speed the cyclotron frequency is equal to =62222222222.22222^{T-1}*. Having a unit as 1\T which is equal to the unit of frequency dimensionally.*

5b. Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space(μ), the current(I), the change in length, the radius, and inversely proportional to square of the radius (r^2*). It can be represented mathematically by.* $\mathbf{dB} = \dfrac{\mu_0}{4\pi} \dfrac{\mathbf{Idl} \times \hat{\boldsymbol{r}}}{r^2}$

Where (μ_0) *is a constant called Permeability of free space* $\mu_0 = 4\pi\times10^{-7}$ T· m/A

The unit of magnetic field is weber\meter square

5b. Magnetic Field of a Straight Current Carrying Conductor

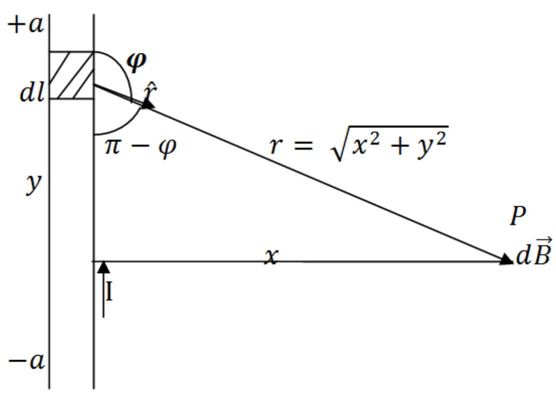


fig 4.2

Fig 4.2: A section of a Straight Current Carrying Conductor

Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$ *B=* $\mathbf{dB} = \dfrac{\mu_0 \mathbf{I}}{4\pi} \int \mathbf{dl} \times \hat{\boldsymbol{r}}$

$$B = \frac{\mu_0}{4\pi}\int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$$

$\sin(\pi - \varphi) = \sin\theta$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From the diagram, $r^2 = x^2 + y^2$ (pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{— (1)}$$

$$\text{But } \sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}} \quad \text{— (2)}$$

Substituting (2) into (1), we have

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \; x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} \, dy$$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} \, dy$$

Using special integrals: $\displaystyle\int \frac{dy}{(x^2+y^2)^{2/3}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{1/2}}$

$$\therefore B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]_{-a}^{a} \quad \text{— (3)}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right) \quad \text{Inputing } a \text{ and } -a \text{ into (3)}$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the distance length ($2a$) of the conductor is very great in comparison to its distance x from point p, we consider it infinitely long. That is when a is very much larger than x, $(x^2+a^2)^{1/2} \simeq a$, as $a \to \infty$

$$B = \frac{\mu_0 I}{2\pi x}$$

with the distance $x = r$ (radius) the magnitude of B =

$$B = \frac{\mu_0 I}{4\pi r} \quad \text{— (4)}$$

Equation defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor.