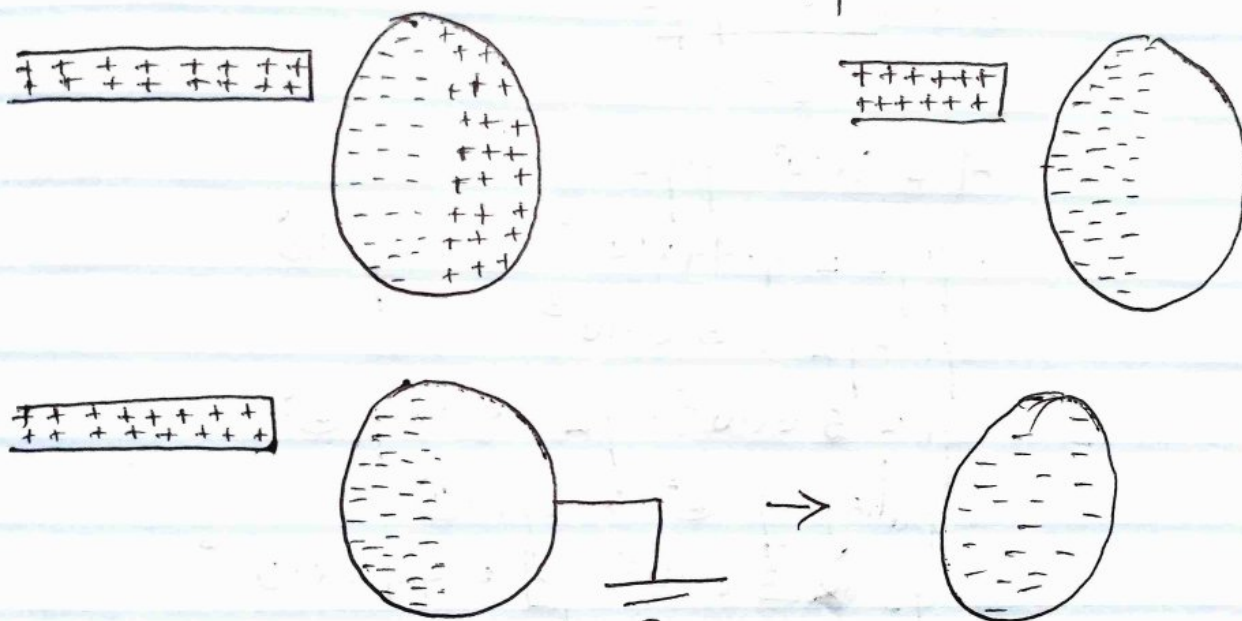


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19/MTHSO1/116

1a Explain with the aid of diagram how you can produce a negatively charged sphere by method of induction.



Electric charges can be obtained on an object without touching it by a process called electrical induction as shown above.

1b Each of 2 small spheres is charged positively, the combined charge being $5.6 \times 10^{-5} \text{ C}$. If each sphere is repelled from the other by a force of 10 N when the spheres are 2.0 m apart, calculate the charge on each sphere.

1b Solution

$$f = 1.0 \text{ N}, \quad r = 2.0 \text{ m}, \quad Q = 5.0 \times 10^{-5} \quad \therefore q_1 + q_2 = Q = 5.0 \times 10^{-5}$$

$$f = \frac{kq_1q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times q_1q_2}{2^2}$$

$$4 = 9 \times 10^9 \times q_1q_2$$

$$\therefore q_1q_2 = 4.44 \times 10^{-10} \quad \dots \text{eqn (1)}$$

$$q_1 + q_2 = 5.0 \times 10^{-5}$$

$$q_1 = 5.0 \times 10^{-5} - q_2 \quad \dots \text{eq (2)}$$

putting eqn (2) in eqn (1)

$$q_2 \times [5.0 \times 10^{-5} - q_2] = 4.44 \times 10^{-10}$$

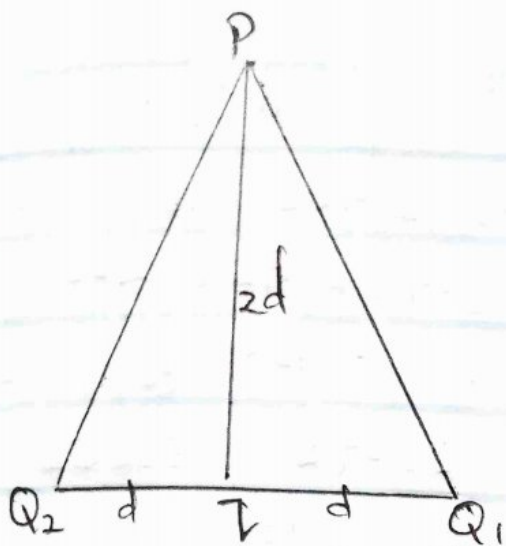
$$5.0 \times 10^{-5} q_2 - q_2^2 = 4.44 \times 10^{-10}$$

$$-q_2^2 + 5.0 \times 10^{-5} q_2 - 4.44 \times 10^{-10} = 0$$

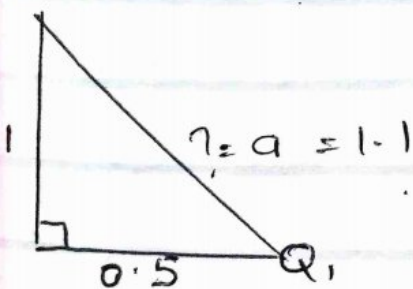
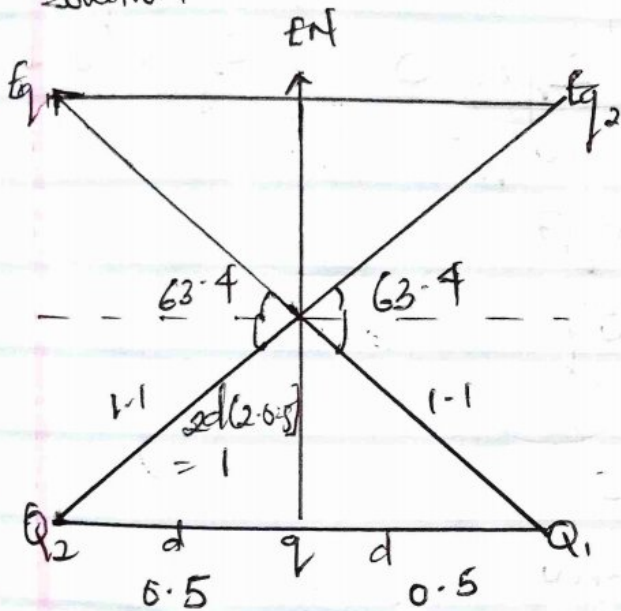
$$q_2 = 3.845 \times 10^{-5} \text{ C} \quad \text{or} \quad q_1 = 5.0 \times 10^{-5} - 1.155 \times 10^{-5} \\ = 1.155 \times 10^{-5} \text{ C} \quad \quad \quad = 3.845 \times 10^{-5} \text{ C}$$

$$\therefore q_2 = 3.845 \times 10^{-5} \text{ C} \quad \text{and} \quad q_1 = 1.155 \times 10^{-5} \text{ C}$$

1c Three charges were positioned as shown in the figure below. If $Q_1 = Q_2 = 8 \mu\text{C}$ and $d = 0.5 \text{ m}$, determine the electric field at $P = 0$



Solution

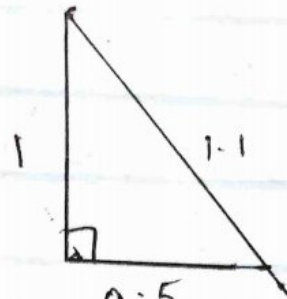


Using Pythagoras theorem

$$a^2 = 1^2 + 0.5^2$$

$$a^2 = 1 + 0.25$$

$$a^2 = 1.25, a = \sqrt{1.25} \therefore a = 1.1$$



$$\tan \theta = \frac{1}{0.5}$$

$$\theta = 63.4$$

$$E_p = E_{q1} + E_{q2} + E_g$$

$$E_{q1} = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.1^2}$$

$$E_{q1} = 59504 \text{ N/C}$$

$$E_{q2} = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.1^2}$$

$$E_{q2} = 59504 \text{ N/C}$$

$$E_g = \frac{kq}{r} = \frac{9 \times 10^9 \times q}{1.1} = 9 \times 10^9 \frac{q}{1.1}$$

$$\therefore E_{q1} = 59504$$

$$E_{q2} = 59504$$

$$E_g = 9 \times 10^9 \frac{q}{1.1}$$

Vector	Angle	X component	Y component
$f_{q1} = 59504$	63.4°	$-59504 \cos 63.4$ $= -26643$	$59504 \sin 63.4$ $= 53205$
$f_{q2} = 59504$	63.4°	$59504 \cos 63.4$ $= 26643$	$59504 \sin 63.4$ $= 53205$
$f_q = 9 \times 10^9 q$	90°	$9 \times 10^9 q \cos 90$ $= 0$	$9 \times 10^9 q \sin 90$ $= 9 \times 10^9 q$
		$\Sigma f_x = 0$	$\Sigma f_y = 106410 + 9 \times 10^9 q$

$$\Sigma p = \sqrt{0^2 + (106410 + 9 \times 10^9 q)^2}$$

$$\Sigma p = \sqrt{(106410 + 9 \times 10^9 q)^2}$$

$$\Sigma p = 106410 + 9 \times 10^9 q$$

$$\text{at } \Sigma p = 0$$

$$106410 + 9 \times 10^9 q = 0$$

$$\frac{9 \times 10^9 q}{9 \times 10^9} = \frac{-106410}{9 \times 10^9}$$

$$q = -1.18 \times 10^{-5} \text{ C}$$

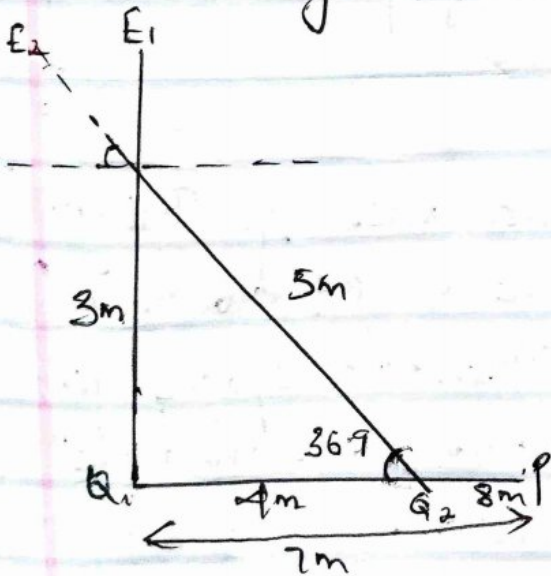
$$q \approx 12 \mu\text{C}$$

2a Distinguish between the terms :- electric field and electric field intensity.

Electric field :- It is a region of space in which an electric charge will experience an electric force while Electric field intensity can be defined as the force per unit charge.

b A positive charge $Q_1 = 8 \text{ nC}$ is at the origin and a second positive charge $Q_2 = 12 \text{ nC}$ is on the x-axis at $x = 4 \text{ m}$.

- The net electric field at a point P on the x-axis at $x = 7 \text{ m}$.
- The electric field at point Q on the y-axis at $y = 3 \text{ m}$ due to the charges



$$\tan \theta = 3/4$$

$$\theta = \tan^{-1} 0.75 \therefore \theta = 36.9^\circ$$

$$E_p = \sum Q_1 + \sum Q_2$$

$$\textcircled{1} \sum Q_1 = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.469 \text{ N/C}$$

$$\sum Q_2 = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 \text{ N/C}$$

$$\sum p_{\text{net}} = 1.469 + 12 = 13.5 \text{ N/C}$$

$$\textcircled{2} \sum_{\text{net}} Q = \vec{E}_1 + \vec{E}_2$$

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

Vector	Angle	X component	Y component
$E_1 = 8 \text{ N/C}$	90°	$8 \cos 90 = 0$	$8 \sin 90 = 8$
$E_2 = 4.82 \text{ N/C}$	36.9°	$= 4.32 \cos 36.9 =$ $= -3.45$	$4.32 \sin 36.9 =$ 2.59
		$\Sigma f_x = -3.45$	$\Sigma f_y = 10.59$

$$\Sigma_{\text{net}} \vec{E} = \sqrt{(-3.45)^2 + (10.59)^2}$$

$$= 11.14 \text{ N/C}$$



4a Magnetic flux is defined as the strength of the magnetic field represented by lines of force. It is represented by the symbol Φ

4b An electron with a rest mass of $9.11 \times 10^{-31} \text{ kg}$ moves in a circular orbit of radius $0.4 \times 10^{-7} \text{ m}$ in a uniform magnetic field of $3.5 \times 10^{-2} \text{ Weber/meter square}$ perpendicular to the speed with which electron moves. Find the cyclotron frequency of the moving electron.

4b Solution

$$M = 9.11 \times 10^{-31} \text{ kg}, \quad r = 1.4 \times 10^{-7} \text{ m}, \quad B = 3.5 \times 10^{-1} \text{ weber/m}^2$$

$$\theta = 90^\circ, \quad w = ? , \quad q = -1.60 \times 10^{-19} \text{ C}$$

$$w = \frac{qB}{m_e}$$

$$w = \frac{-1.60 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = -6.15 \times 10^{10} \text{ rad/sec}$$

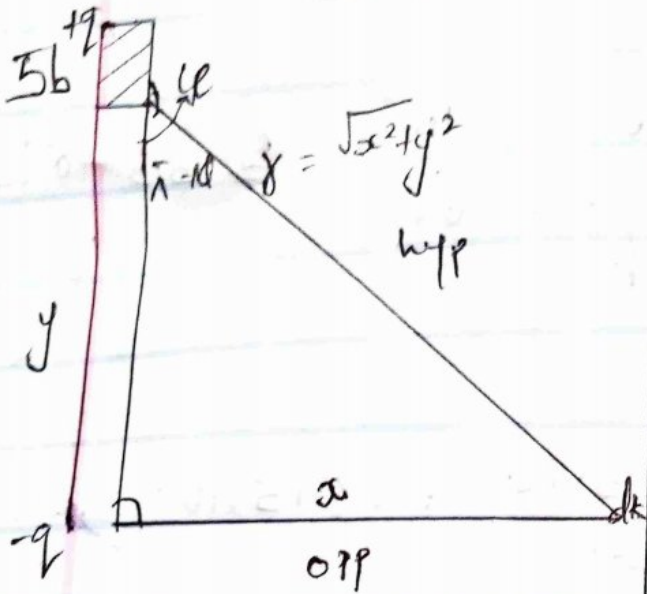
4c Discuss your answer in the above!

Since our cyclotron frequency is negative, -6.15×10^{10} rad/sec. It means that the charge particle electron circulates in a negative or opposite direction at the angular frequency.

5a State the Biot-Savart law

Biot-Savart law is an equation that describes the magnetic field created by a current-carrying wire and allows you to calculate its strength at various forms.

5b Using the Biot-Savart law, show that the magnitude of the magnetic field of a straight current carrying conductor is given as $B = \frac{\mu_0 I}{2\pi r}$



Applying the Biot-Savart law, we find the magnitude of field dB

$$B = \frac{\mu_0}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

from the diagram, $r^2 = x^2 + y^2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \dots \text{--- (1)}$$

$$\text{but } \sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}}$$

substituting eqn 2 into 1 we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl}{\sqrt{x} \sqrt{(x^2 + y^2)} (x^2 + y^2)^{1/2}}$$

Using law of indices

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl}{\sqrt{x} (x^2 + y^2)^{3/2}}$$

recall $dl = dy$

$$\text{so } B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dy}{\sqrt{x} (x^2 + y^2)^{3/2}} \dots \text{--- (2)}$$

using special integrals

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

equation 3 therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

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∴ When the length $2a$ of the conductor is very great in comparison to its distance from point p , we consider it ~~as~~ infinitely long. That is, when a is much larger than x .

$$\therefore (x^2 + a^2)^{1/2} = a \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi a}$$

In a physical situation, we have axial symmetry about the y -axis. Thus at all points in a circle of radius r , around the conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r}$$