EZENKWATA VICTOR -C.
17/ENG06/026
Elect/ Elect
EEE 316

4a) From $\frac{d^2 E_y}{dx^2} = (j\omega\mu\varepsilon - \omega^2\mu\varepsilon)E_y$

$\Rightarrow \frac{d^2 E_y}{dx^2} = \gamma^2 E_y$

$\gamma = \alpha + j\beta$

$E_y = E_o e^{-\gamma x} = E_o e^{-\alpha x} e^{-j\beta x}$

$\frac{d^2 E_y}{dx^2} = j\omega\mu\sigma E_y = \gamma^2 E_y$

$\gamma^2 = j\omega\mu\sigma$

$\gamma = \sqrt{j\omega\mu\sigma} = \alpha + j\beta$

from $\sqrt{j} = \frac{1+j}{\sqrt{2}}$

$\therefore \gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}}$

$\rightarrow \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$ and $\beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

$$E_y = E_0 e^{-\sqrt{\frac{\omega\mu\sigma}{2}}x} e^{-j\sqrt{\frac{\omega\mu\sigma}{2}}x}$$

we have;- $E_y = E_0 e^{-x/\sigma} e^{-jx/\sigma}$

It shows that the amplitude of the wave decreases exponentially as it penetrates a conducting medium by a factor $e^{-x/\sigma}$.

4b) Skin depth is defined as the depth of penetration of a wave inside a conductor

$$\sigma = \sqrt{\frac{2}{\omega\mu\sigma}}$$

and $\omega = 2\pi f$

$$= \sqrt{\frac{2}{2\pi f \omega\sigma}} = \sqrt{\frac{1}{\pi f \mu\sigma}}$$

$$\therefore \sigma = \sqrt{\frac{2}{\omega\mu\sigma}} \text{ or } \sqrt{\frac{1}{\pi f \mu\sigma}}$$

4c) $f = 10\text{MHz} \simeq 1\times10^{7}\text{Hz}$

$\sigma = 5.8\times10^{7}$ S/m, $\mu = 1$, $\mu_0 = 1.257\times1$

$$\sigma = \sqrt{\frac{1}{\pi f \mu\sigma}} = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

$\therefore L = 2.90 \times 10^{-7}$ H/m

c) characteristic Impedance, Z

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{2.90 \times 10^{-7}}{3.84 \times 10^{-11}}}$$

$$Z_0 = 86.90\,\Omega$$

d) Phase velocity, $V_p$

$$V_p = \frac{1}{\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{(2.9 \times 10^{-7}) \times (3.84 \times 10^{-11})}}$$

$$V_p = 299664563.4$$

$$= 3 \times 10^8$$

$\mu = \mu_r \times \mu_0 = 1 \times 1.257 \times 10^{-6}$

$= 1.257 \times 10^{-6}$

$$\sigma = \frac{1}{\sqrt{\pi \times 1 \times 10^{7} \times 1.25 \times 10^{-6} \times 5.8 \times 10^{7}}}$$

$\sigma = 2.09 \times 10^{-5}\,m.$

7. $b = 10mm = 0.01m$, $a = 3mm = 0.003m$

$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

$\mu_0 = 1.257 \times 10^{-6}\ H/m$

a. Capacitance per meter, c

$$c = \frac{2\pi \varepsilon_0}{\log_e b/a}$$

$$c = \frac{2\pi \times 8.85 \times 10^{-12}}{\log_e \left(\frac{0.01}{0.03}\right)}$$

$c = 3.84 \times 10^{-11}\ f/m$

b. Inductance per meter, L

$$L = \frac{\mu_0}{2\pi} \log e^{b/a}$$

$$L = \frac{1.257 \times 10^{-6}}{2\pi} \cdot \log e \frac{0.01}{0.03}$$

$$L = \frac{1.257 \times 10^{-6}}{2\pi} \cdot \log 28.03$$