PHY 102

BILIAMEEN ADEDOLAPO ABDULFATTAH
MECHATRONICS ENGINEERING
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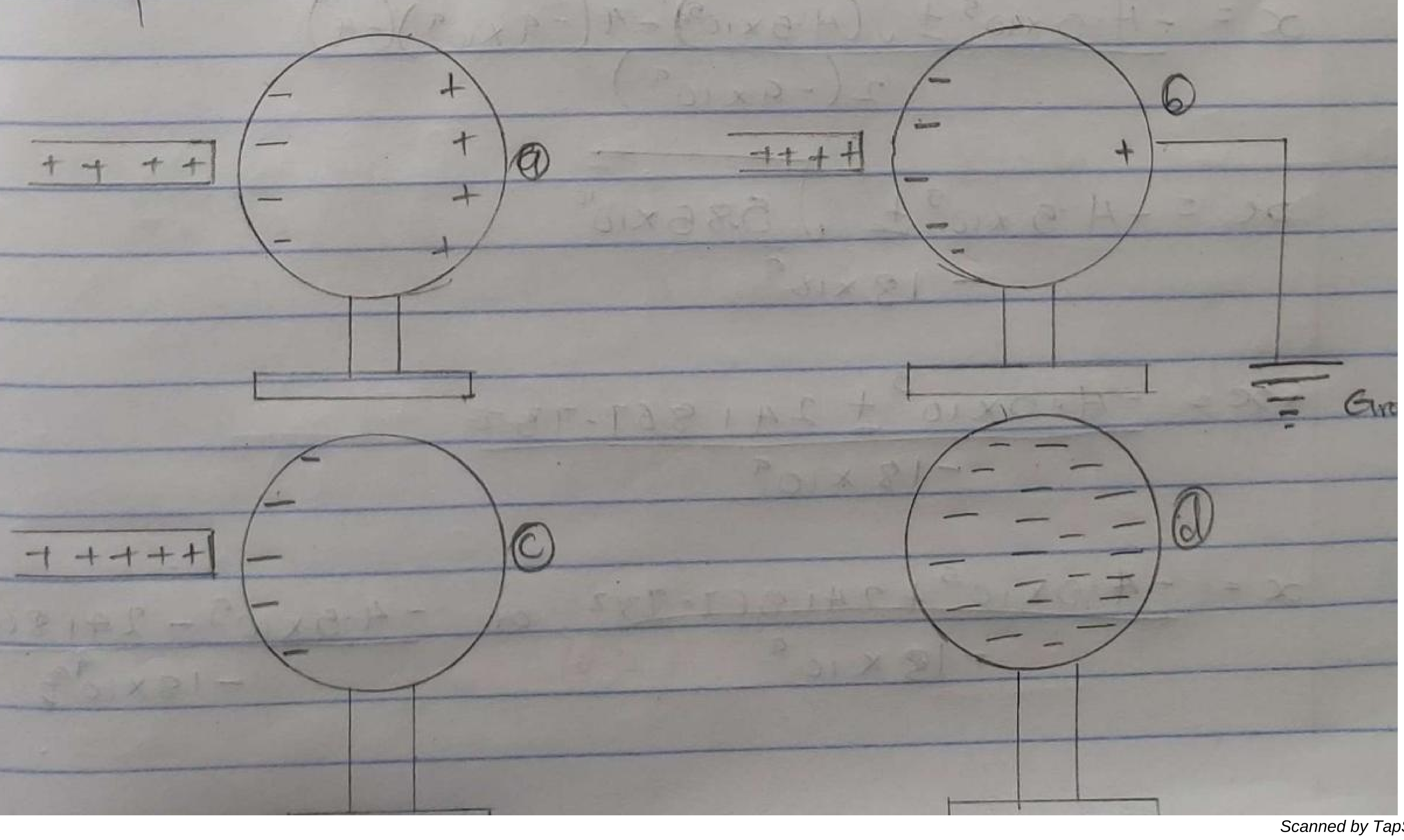
1a To produce a negatively charged sphere by method of induction we consider a positively charged rubber rod broug near a neutral (uncharged) conducting sphere that is insulate So that there is no conducting path to the ground. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sp so that some protons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to the positively charged rod has an excess of negative charge b of the migration of protons away from this location. If a groun conducting wire is then connected to the sphere some of the protons leave the sphere and travel to the earth. If the w to the ground is then removed the conducting sphere is left wit an excess of induced negative charge.

Finally, when the rubber rod is removed from the vicinity o the sphere the induced negative charge remains on the ungro sphere and becomes uniformly distributed over the surface o the sphere.



Each of two small spheres is charged positively, their combined charge being $5.0 \times 10^{-5}C$. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart. Calculate the charge on each sphere.

Solution.

$q_1 + q_2 = 5.0 \times 10^{-5}$ $F = 1N$ $r = 2m$

$q_1 = 5.0 \times 10^{-5} - q_2$

$f = \frac{Kq_1q_2}{r^2}$; $1 = \frac{9 \times 10^9 \times (5.0 \times 10^{-5} - q_2)q_2}{2^2}$

$1 = \frac{9 \times 10^9 \times (5.0 \times 10^{-5} - q_2)q_2}{4}$

$4 = 9 \times 10^9 \times (5.0 \times 10^{-5} - q_2)q_2$

$4 = (4.5 \times 10^5 - 9 \times 10^9 q_2)q_2$

$4 = 4.5 \times 10^5 q_2 - 9 \times 10^9 q_2^2$

$-9 \times 10^9 q_2^2 + 4.5 \times 10^5 q_2 - 4 = 0$

Using quadratic formulae $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-4.5 \times 10^5 \pm \sqrt{(4.5 \times 10^5)^2 - 4(-9 \times 10^9)(-4)}}{2(-9 \times 10^9)}$

$x = \frac{-4.5 \times 10^5 \pm \sqrt{5.85 \times 10^{10}}}{-18 \times 10^9}$

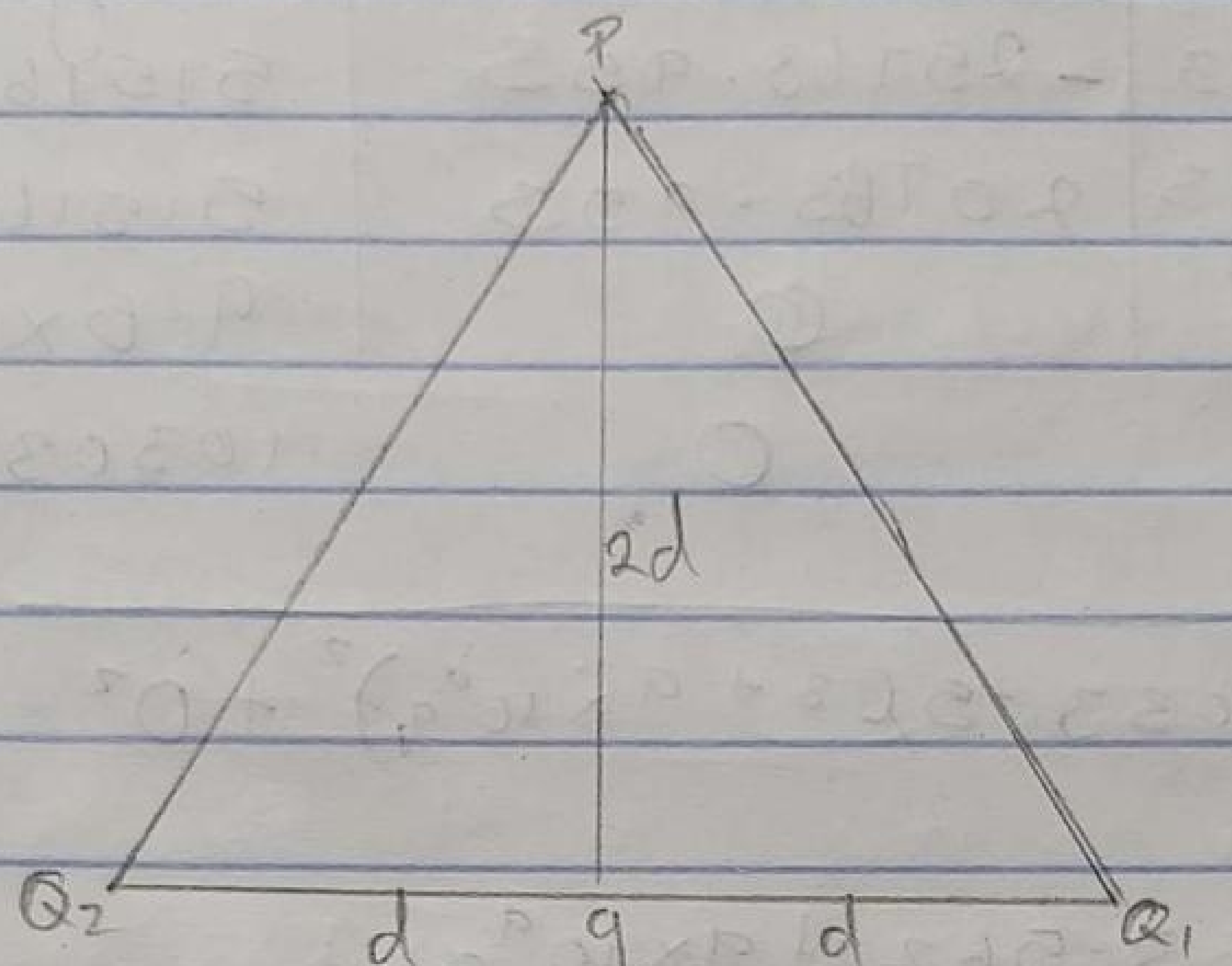
$x = \frac{-4.5 \times 10^5 \pm 241867.732}{-18 \times 10^9}$

$x = \frac{-4.5 \times 10^5 + 241867.732}{-18 \times 10^9}$ or $\frac{-4.5 \times 10^5 - 241867.732}{-18 \times 10^9}$

$x = 1.15629 \times 10^{-5} C$ or $3.8437 \times 10^{-5} C$

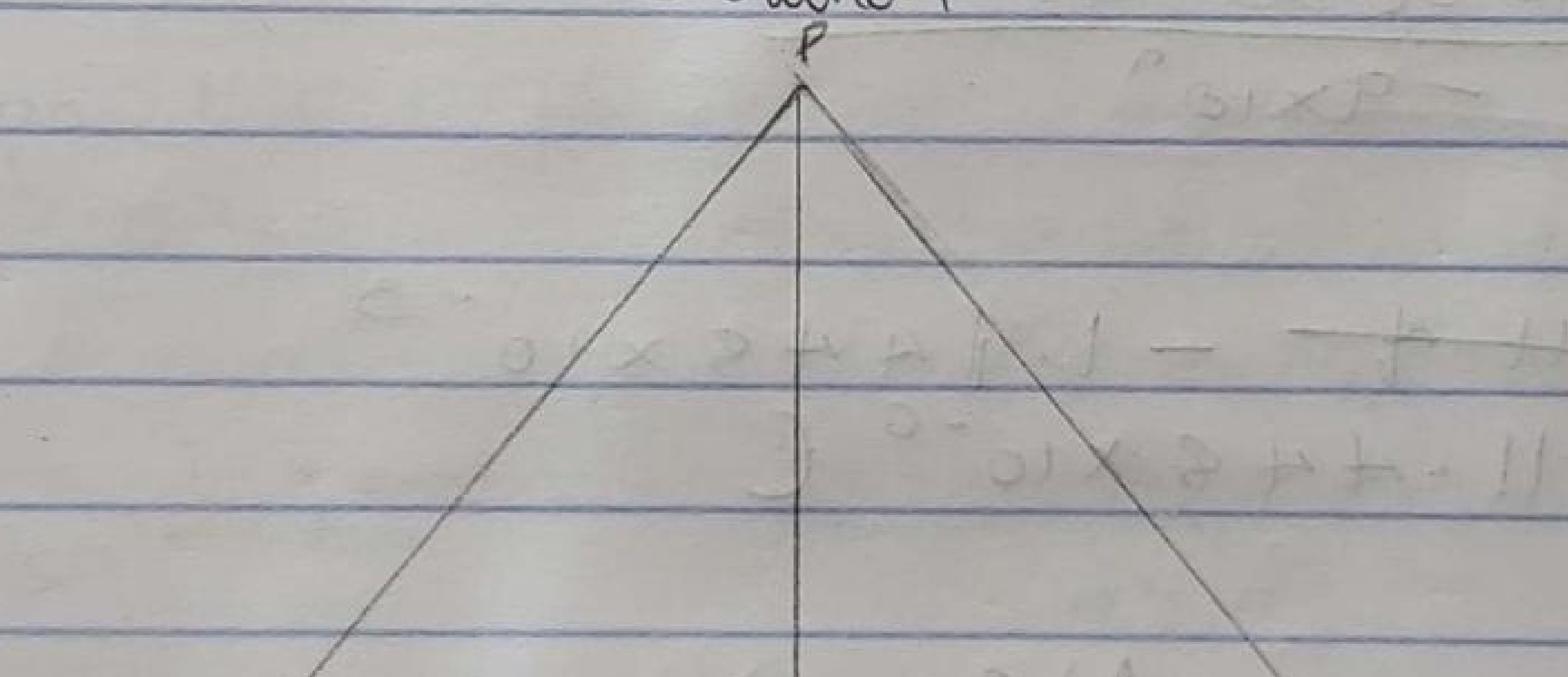
$q_1 = 1.16 \times 10^{-5} C$ and $q_2 = 3.84 \times 10^{-5} C$

1c



Three charges were positiond as shown above if $Q_1 = Q_2 = 8\mu C$ and $d = 0.5m$ determine q if the electric field at p is zero

Solution



$(2d)^2 + d^2 = 5d^2$ $\qquad \sin\theta = \frac{2d}{d\sqrt{5}} = \frac{2}{\sqrt{5}}$

$\sqrt{5d^2} = d\sqrt{5}$

$\theta = \sin^{-1} 2/\sqrt{5} = 63.43°$ $\quad d = 0.5m$

$E_1 = \frac{KQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(\sqrt{5}/2)^2} = 57600 NC^{-1}$

$E_2 = \frac{KQ_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(\sqrt{5}/2)^2} = 57600 NC^{-1}$

$$E_q = \frac{Kq}{r^2} = \frac{9\times10^9\times q}{1^2} = 9\times10^9 q$$

E	θ	$E_x = E\cos\theta$	$E_y = E\sin\theta$
57600	63.43	−25763.953	51516.781
57600	63.43	25763.953	51516.781
$9\times10^9 q$	90°	0	$9.0\times10^9 q$
=		0	$103033.562 + 9\times10^9 q$

$$0 = \sqrt{(103033.562 + 9\times10^9 q)^2 + 0^2}$$

$$0 = 103033.562 + 9\times10^9 q$$

$$-9\times10^9 q = 103033.562$$

$$q = \frac{103033.562}{-9\times10^9}$$

$$q = \cancel{11.4} \ -1.1448\times10^{-5}$$

$$q = 11.448\times10^{-6}\ C$$

$$q = 11.448\ \mu C$$

2a Electric field	Electric field intensity
An electric field is a region in space in which an electric Charge will experience an electric force.	The electric field intensity E is the force per unit charge. Mathematically $E = \frac{F(N)}{q_0(C)}$ where F is force in Newtons and q_0 is a unit charge in Coulumb. It is measured in Newton per Coulumb N/C or NC^{-1}

b A positive charge $Q_1 = 8nC$ is at the origin and a second positive charge $Q_2 = 12nC$ is on the x-axis at $x = 4m$

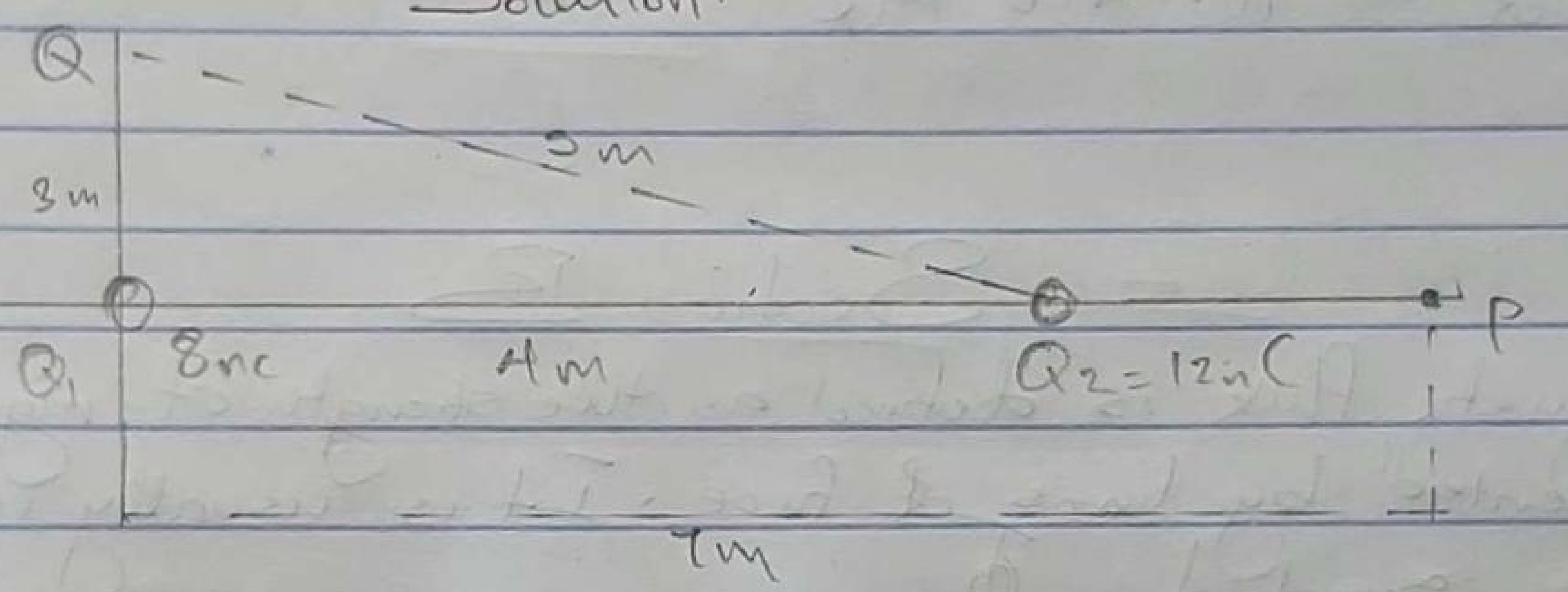
Find

i) The net electric field at a point P on the x axis at $x = 7m$

ii) The electric field at a point Q on the y axis at $y = 3m$ due to the charges.

Solution.

i)



$\sin\theta = \frac{3}{5}$ $\theta = \sin^{-1}(3/5)$ $\theta = 36.87$

At point P

$$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^{9} \times 8\times10^{-9}}{7^2} = 1.46939\ NC^{-1}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^{9} \times 12\times10^{-9}}{3^2} = 12\ NC^{-1}$$

$$E_{net} = E_1 + E_2 = 12 + 1.46939 = 13.46939\ NC^{-1}$$

ii) $$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^{9} \times 8\times10^{-9}}{3^2} = 8\ NC^{-1}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^{9} \times 12\times10^{-9}}{5^2} = 4.32\ NC^{-1}$$

E	θ	$E_x = E\cos\theta$	$E_y = E\sin\theta$
8	90°	0	+8
4.32	36.87	3.456	2.592
=		$3.456\ NC^{-1}$	$10.592\ NC^{-1}$

$E_{net} = \sqrt{E_x^2 + E_y^2}$

$E_{net} = \sqrt{(3.496)^2 + (10.592)^2}$

$E_{net} = \sqrt{124.1344}$

$E_{net} = 11.1416\ NC^{-1}$

Section B

4a Magnetic flux is defined as the strength of magnetic field represented by lines of force. It is usually represented by the symbol Φ

b An electron with a rest mass of 9.11×10^{-31} kg moves in a circular orbit of radius 1.4×10^{-7} m in a uniform magnetic field of 3.5×10^{-4} weber/meter square perpendicular to the speed with which electron moves. Find the cyclotron frequency of the moving electron.

Solution

$\frac{Mv^2}{r} = qvB \qquad \omega = v/r$

so $m\cancel{v}\omega = q\cancel{v}B$

$m\omega = qB$

$\omega = \frac{qB}{M}$

$\omega = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$

$\omega = 6.14709 \times 10^{10}\ rads^{-1}$

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c Discussing my answer above, I would say If an electron with a rest mass of 9.11×10^{-31} kg and charge 1.6×10^{-19} C is in motion in a magnetic field of 3.5×10^{-4} Tesla perpendicular with the field It will have an angular frequency of 6.14709×10^{10} rads^{-1}

5 The Biot Savart Law is based on the following observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\hat{l}$ of a wire carrying a steady current I

i) The vector $d\vec{B}$ is perpendicular both to $d\hat{l}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\hat{l}$ towards P

ii) The magnitude of dB is inversely proportional to r^2, where r is the distance from dl to P

iii) The magnitude of dB is proportional to the current I and to the magnitude of the length element $d\vec{l}$

iv) The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where the angle between $\hat{r}$ and dl.

Mathematically

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\hat{l} \times \hat{r}}{r^2}$$

Where $d\vec{B}$ = Magnetic field vector in Tesla
I = Current.

5b Using the Biot-Savart Law to show that the magnitude of the magnitude of magnetic field of a straight current carrying conductor is given as

$$B = \frac{\mu_0 I}{2\pi r}$$

From the mathematical expression of Biot Savart law we have

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\hat{l} \times \hat{r}}{r^2}$$

$$\int d\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

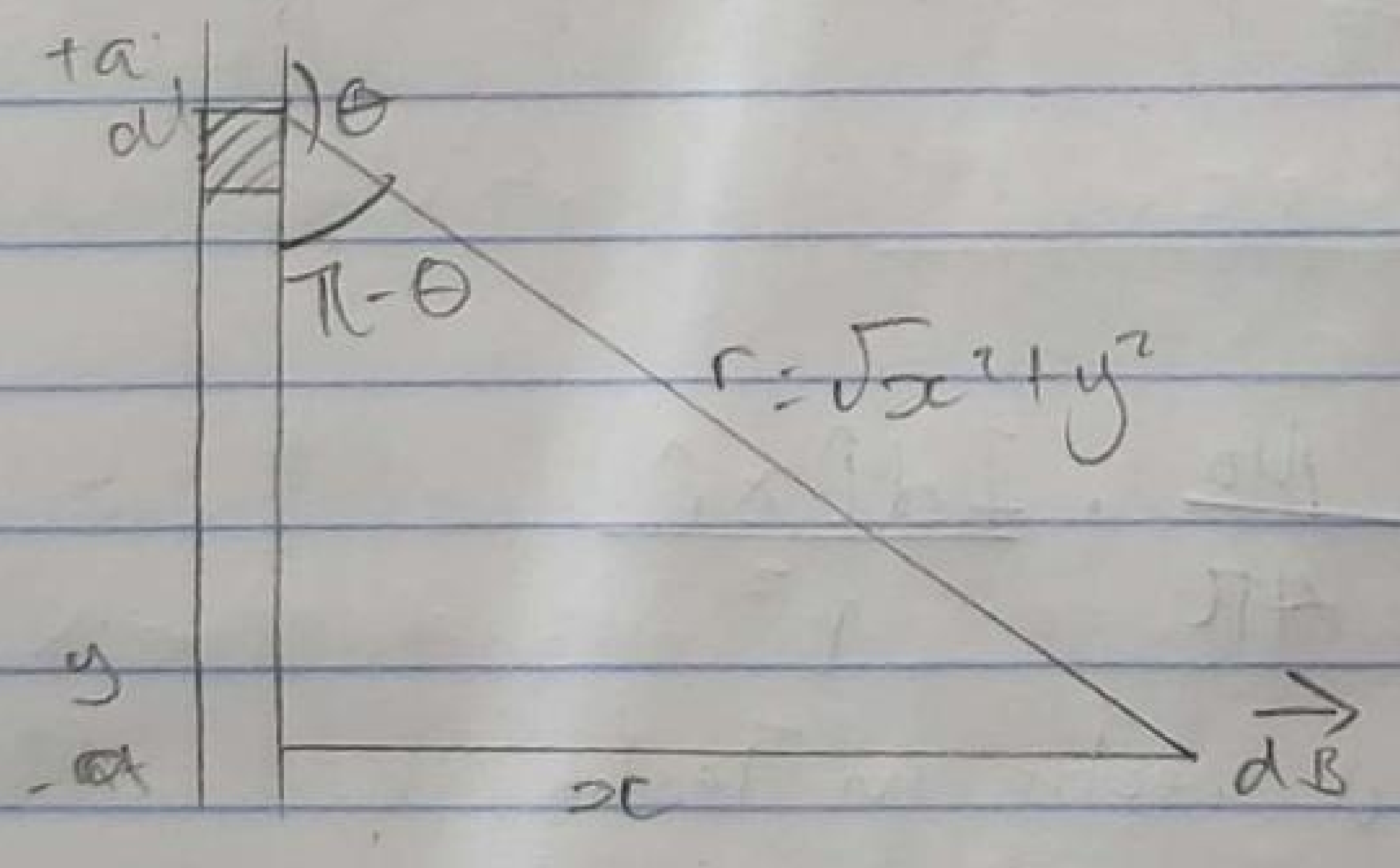
$\vec{dl} \times \hat{r} = dl\, r \sin\theta$

$r =$ unit vector $= 1$

$dl \sin\theta$

$\therefore$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin\theta}{r^2}$$



$r = \sqrt{x^2+y^2}$

$r^2 = x^2+y^2$

$\theta = \pi - \theta$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi-\theta)}{r^2}$$

$\sin(\pi-\theta) = x/r$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl}{r^2} \cdot \frac{x}{r}$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{dl}{(x^2+y^2) \times (x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{dl}{(x^2+y^2)^{3/2}}$$

$$dl \simeq dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} \, dy$$

$$B = \frac{\mu_0 I x}{4\pi} \frac{1}{x^2} \cdot \frac{y}{(x^2+y^2)^{1/2}} \Bigg|_{-a}^{a}$$

$$B = \mu_0 I x \cdot \frac{a}{x^2(x^2+a^2)^{1/2}} - \frac{-a}{x^2(x^2+(-a)^2)^{1/2}}$$

$$B = \frac{\mu_0 I x}{4\pi} \cdot \frac{2a}{x^2(x^2+a^2)^{1/2}}$$

$$(x^2+a^2)^{1/2} \simeq a, \text{ as } a \to \infty$$

$$a >> x$$

$$B = \frac{\mu_0 I}{4\pi x} \cdot \frac{2a}{a}$$

$$B = \frac{\mu_0 I}{2\pi x} \qquad x = r \therefore$$

$$B = \frac{\mu_0 I}{2\pi r}$$