Name: Umar Abdullahi Dabarako

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MAT 104 Assignment 07 MBBS

1 $\int \frac{2x}{\sqrt{4x^2-1}}dx = \frac{1}{2}\sqrt{4x^2-1} + C$

Solution

$\int \frac{2x}{4x^2-1} \cdot dx$

Let $u = \sqrt{4x^2-1}$, $p = 4x^2-1$

$\therefore u = \sqrt{p}$

If $p = 4x^2-1$ $\quad u = \sqrt{p}$

$\frac{dp}{dx} = 8x$ $\quad \frac{du}{dp} = \frac{1}{2\sqrt{p}} = \frac{1}{2\sqrt{4x^2-1}}$

$\frac{du}{dx} = \frac{dp}{dx} \times \frac{du}{dp}$

$\frac{du}{dx} = \cancel{8}^{4}x \times \frac{1}{\cancel{2}_{1}\sqrt{4x^2-1}}$

$\frac{du}{dx} = \frac{4x}{\sqrt{4x^2-1}}$

$dx = \frac{\sqrt{4x^2-1} \cdot du}{4x}$

$\int \frac{\cancel{2x}^{1} \cdot \cancel{\sqrt{4x^2-1}} \cdot du}{\cancel{\sqrt{4x^2-1}} \cdot \cancel{4x}_{2}}$

$\int \frac{1}{2} du$

$\frac{1}{2}\int du = \frac{1}{2}u + C$

$u = \sqrt{4x^2-1}$

$\therefore \int \frac{2x}{\sqrt{4x^2-1}}dx = \frac{1}{2}\sqrt{4x^2-1} + C$

2) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx = \frac{\cancel{(\sin^{-1})}(\sin^{-1}x)^2}{2} + C$

Solution

$\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

Let $u = \sqrt{1-x^2}$ and $p = 1-x^2$

$u = \sqrt{p}$

$\frac{du}{dp} = \frac{p^{-1/2}}{2}$ $\quad \frac{dp}{dx} = -2x$

$\frac{du}{dx} = \frac{dp}{dx} \times \frac{du}{dp}$

$\frac{du}{dx} = -\cancel{2}x \times \frac{1}{\cancel{2}_{1}} \times \frac{1}{\sqrt{p}}$

$\frac{du}{dx} = \frac{-x}{\sqrt{p}} = \frac{-x}{\sqrt{1-x^2}}$

$dx = -\frac{\sqrt{1-x^2} \cdot du}{x}$

$\therefore \int \frac{\sin^{-1}x}{\cancel{\sqrt{1-x^2}}} \cdot -\frac{\cancel{\sqrt{1-x^2}}}{x} \cdot du$

$-\int \frac{\sin^{-1}x \, du}{x}$

If $u = \sqrt{1-x}$

$x = 1-u^2$

$\therefore \int \frac{\sin^{-1}(1-u^2)}{1-u^2} du = \ln(\sin^{-1}x) + C$

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3) $\int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + C$ ANS

$\left\{ \begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \\ u^6 du = \frac{u^7}{7} \end{array} \right\}$

Solution

$\int (\tan x)^6 \sec^2 x \, dx$

Let $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x \, dx$

$\frac{dx}{1} = \cancel{\sec^2} \frac{du}{\sec^2 x}$

$\int u^6 \times \cancel{\sec^2 x} \times \frac{du}{\cancel{\sec^2 x}}$

$\int u^6 du = \frac{u^7}{7} + C$

$\int (\tan x)^6 \sec^2 x \, dx = \frac{(\tan x)^7}{7} + C$