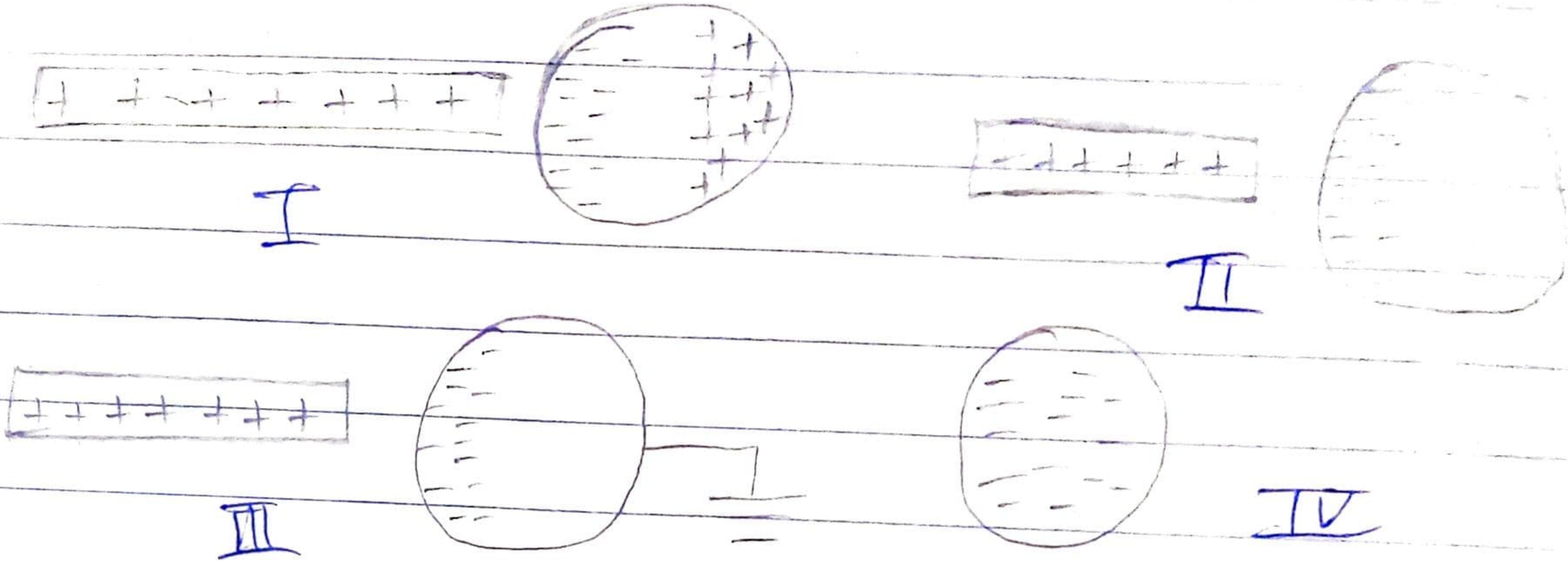
BUKOLA OLUWARANTIMI
19/ENG05/020
MECHATRONICS ENGINEERING



Consider a positively charged rubber rod brought near a neutral conducting sphere ~~that~~ is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere in I. The section closer to the positive rod is negatively charged (II). If the grounded conducting wire is connected to the rod, some electrons leave the sphere & travel to the earth (III). Finally the rubber rod is removed from the vicinity and the induced negative charge becomes evenly distributed on the sphere.

$q_1 = ?$ $q_2 = ?$ $q_1 + q_2 = 5.0 \times 10^{-5}$

$r = 2\,m$ $F = 1N$ $K = 8.9875 \times 10^{9}\,Nm^2$

$\approx 9 \times 10^{9}\,Nm^2/C$

$$F = \frac{K q_1 q_2}{r^2} \therefore 1 = \frac{9 \times 10^{9} \times q_1 \times q_2}{2^2}$$

$$\frac{4}{9 \times 10^{9}} = q_1 q_2 = 4.4 \times 10^{-10} \quad \text{—(1)}$$

$$q_1 = 5.0 \times 10^{-5} - q_2 \quad \text{—(2)}$$

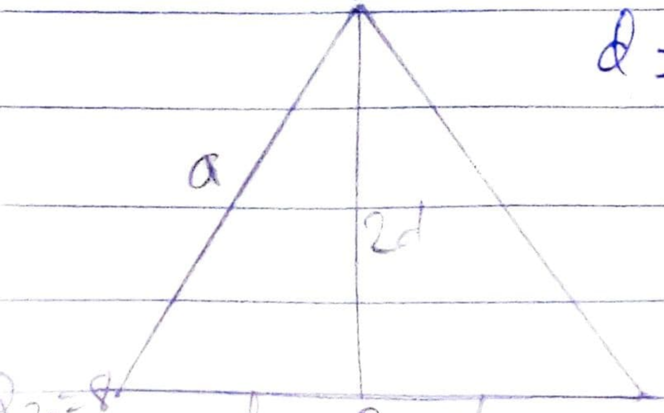
By substituting $(5 \times 10^{-5} - q_2)\, q_2 = 4.4 \times 10^{-10}$

$$-q_2^2 + 5 \times 10^{-5} q = 4.4 \times 10^{-10}$$

$$q_2^2 - 5 \times 10^{-5} q + 4.4 \times 10^{-10} = 0$$

$q_1 = 3.86 \times 10^{-5}$ $q_2 = 1.14 \times 10^{-5}$

By using quadratic equation.



$d = 0.5m$ $2d = 2 \times 0.5$

Using pythagoras

$$a^2 = d^2 + (2d)^2$$

$$a = \sqrt{0.5^2 + 1^2}$$

$$a = 1.118\,m$$

1C $Q_1 = Q_2 = 8\mu C$

$d = 0.5m$

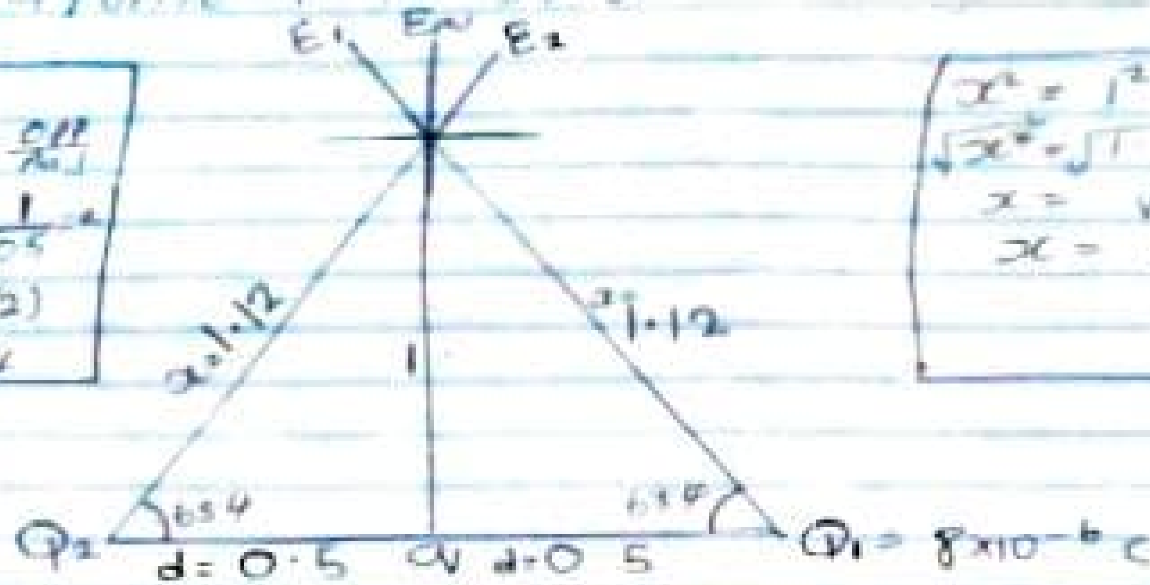
determine Q if electric Field at a point P is zero

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{1}{0.5}$

$\theta = \tan^{-1}(2)$

$\theta = 63.4$



$x^2 = 1^2 + 0.5^2$

$\sqrt{x^2} = \sqrt{1.25}$

$x = \sqrt{1.25}$

$x = 1.12$

$$E_1 = \frac{kQ_1}{r_1^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2} = 5739.795918$$

$$E_2 = \frac{kQ_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{1.12^2} = 5739.795918$$

$$E_Q = \frac{kQ}{r^2} = \frac{9\times10^9 \times Q}{1} = 9\times10^9 Q$$

Vector	angle	x-comp	y-comp
$E_1 = 5739.795918$	$63.4°$ $E_1 =$	$E_1 \times \cos\theta$ -2570.045785	~~$E_1 \times \sin\theta$~~ 5132.262839
$E_2 = 5739.795918$	$63.4°$	2570.045785	5132.262839
$E_Q = 9\times10^9 Q$	$90°$	$E_Q\cos\theta = 0$ $\Sigma x = 0$	$9\times10^9 Q$ Σy 10264.525678

$$\text{magnitude} = \sqrt{(\Sigma_x)^2 + (\Sigma_y)^2}$$

$$E_q = \sqrt{(0)^2 + (10264.52568)^2}$$

since $E_q = 0$

$$0 = 9\times10^9 q + 10264.52568$$

making q subject of formulae

$$q = -\frac{10264.52568}{9\times10^9}$$

$$q = 1.1405028 53\times10^{-6}$$

$$\Rightarrow q = 11.4\,C$$

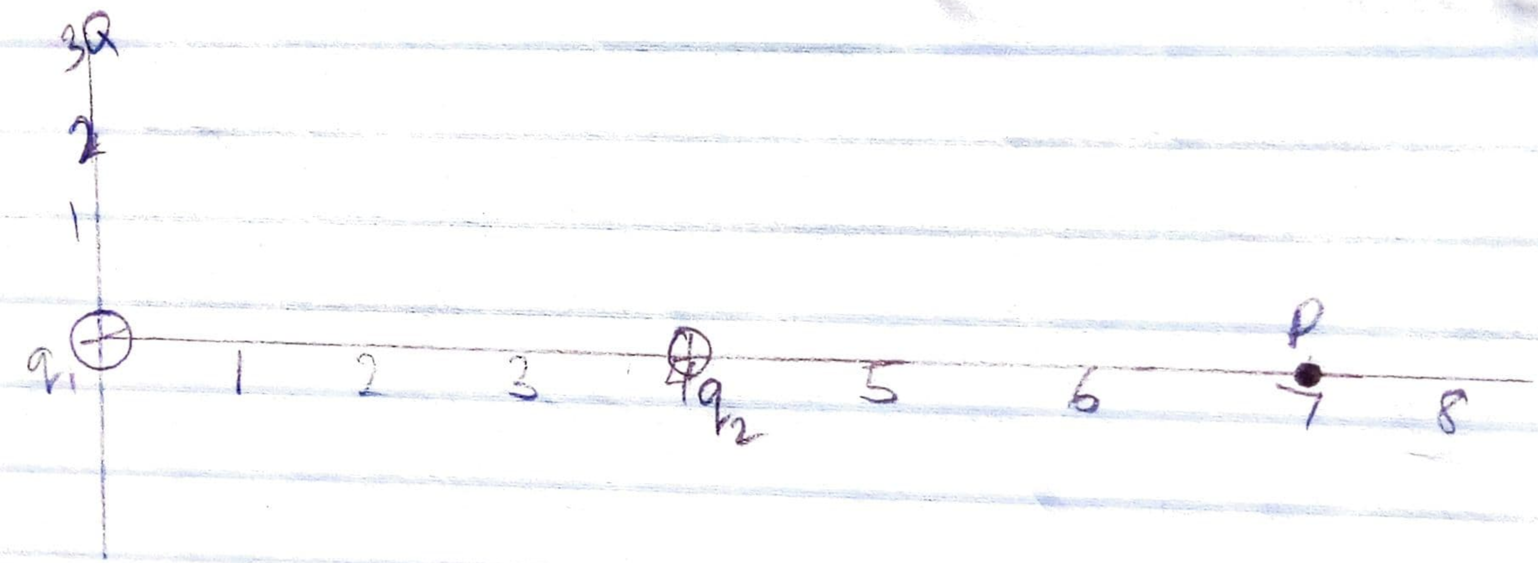
.

(i) **Volume charge density,** $\rho = \frac{dQ}{dV}$'n $dQ = \rho dV$

(ii) **Surface charge density,** $\sigma = \frac{dQ}{dA}$'n $dQ = \sigma dA$

(iii) **Linear charge density,** $\lambda = \frac{dQ}{dL}$'n $dQ = \lambda dL$

① Electric field is the region or space around which an electric charge will experience an electric force while electric field intensity is the force per unit charge on electron.



at $x = 7m$ $\qquad q_1 = 8\times10^{-9}$

$K = 9\times10^{9}$ $\qquad q_2 = 12\times10^{-9}$

$$E_{net} = \left(\frac{9\times10^{9}\times8\times10^{-9}}{7^2}\right) + \left(\frac{9\times10^{9}\times12\times10^{-9}}{3^2}\right)$$

$$E_{net} = 1.47 + 12$$

$$E_{net} = 13.47 = 13.5\,N/C$$

3 (a) The continuous charged distribution is divided into infinitesimal charge disk contained in a volume element dV or area element dA, or length element

i volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

ii Area (Surface) charge density, $\sigma = dQ/dA \rightarrow dQ = \sigma dA$

iii Linear charge density, $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

(b)

$$dW = F \cdot dL$$

But

$$F = -q_0 E$$

$$dW = -q_0 E \, dL$$

$$\text{Total } W = -q_0 \int_A^B E \, dL$$

$$V_B - V_A = \frac{W(A \rightarrow B)_{AB}}{q_0}$$

$$\therefore V_B - V_A = -\int_A^B E \, dL$$

Electric potential Difference is work done per unit charge

4. (a) Magnetic flux is the measure of the total magnetic field that passes through an area

moves in a circular orbit of radius 1·4 $\times 10^{-7}$ m in a uniform magnetic field of 3.5×10^{-1} telsa, perpendicular to the speed of light with which the electron moves. Find the cyclotron frequency of the moving electron

Solution

$m = 9.11 \times 10^{-31}$ $\theta = 90^{\circ}$

$r = 1.4 \times 10^{-7} m$

$B = 3.5 \times 10^{-1}$ telsa

$V = 3 \times 10^{8} m/s$

Cyclotron frequency $= \frac{QB}{2\pi m}$

$\sin\theta QB = \frac{mv}{r}$

$Q = \frac{mv}{rB\sin\theta}$ $\therefore$ $Q = \frac{9.11 \times 10^{-31} \times 3 \times 10}{1.4 \times 10^{-7}}$

$$Q = \frac{9.11 \times 10^{-31} \times 3 \times 10^{8}}{1.4 \times 10^{-7} \times 3.5 \times 10^{-1} \times \sin 90^\circ}$$

$$Q = 5.578 \times 10^{-15}\ C$$

$$\pi = 3.142 = \frac{22}{7}$$

$$f = \frac{QB}{2\pi m}$$

$$f = \frac{5.578 \times 10^{-15} \times 3.5 \times 10^{-1}}{2 \times 3.142 \times 9.11 \times 10^{-31}}$$

$$f = 3.41 \times 10^{14}\ Hz$$

(c) In b, the ~~the~~ electron moved with an angular speed of $3.41 \times 10^{14}\ s^{-1}$

5. Biot-savart law states that the magnetic field is directly proportional to the product permeability of free space(μ),the current(I),the change in length, the radius and inversely proportional to square of radius (r^2). It can be represented mathematically by

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{l}\times \vec{r}}{r^2}$$

whereμ_o is a constant called Permeability of free space.

$$\mu_o = 4\pi\times10^{-7}T.\frac{m}{A}$$

The unit of $\vec{B}$ is weber\metre square

5b. Magnetic Field of a Straight Current Carrying Conductor

Fig 1: A section of a Straight Current Carrying Conductor

Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_o I}{4\pi}\int_a^b \frac{dl\sin\varphi}{r^2}$$

$$\sin(\pi-\varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_o I}{4\pi}\int_a^b \frac{dl\sin(\pi-\varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagorastheorem)

$$B = \frac{\mu_o I}{4\pi}\int_a^b \frac{dl\sin(\pi-\varphi)}{x^2+y^2} \quad (*)$$

$$\text{But}\sin(\pi-\varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}\dots \quad (**)$$

Substituting (**)into (*), we have

$$B = \frac{\mu_o I}{4\pi}\int_a^b dl\frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_o I}{4\pi}\int_a^b dl\frac{x}{(x^2+y^2)^{3/2}}$$

magnitude of the field dB

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagorastheorem)

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi-\varphi)}{x^2 + y^2} \dots \quad (*)$$

$$\text{But} \sin(\pi-\varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \dots \quad (**)$$

Substituting (**)into (*), we have

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_o I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2 + y^2)^{3/2}} dy \dots \quad (***)$$

Using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

Equation (***) therefore becomes

$$B = \frac{\mu_o I x}{4\pi}\left[\frac{y}{x^2(x^2 + y^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_o I x}{4\pi}\left(\frac{2a}{x^2(x^2 + a^2)^{1/2}}\right)$$

$$B = \frac{\mu_o I}{4\pi x}\left(\frac{2a}{(x^2 + a^2)^{1/2}}\right)$$

When the length 2a of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is, when a is much largerthan x,

$$(x^2 + a^2)^{1/2} \equiv a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_o I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y- axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is

$$B = \frac{\mu_o I}{2\pi r} \dots \quad (\#)$$

Equation (#) defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor.