

SECTION A

1a Charging by Induction:

Electric charges can be obtained on an object without touching it by a process called **ELECTROSTATIC INDUCTION**.

Consider a positively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to the ground as shown below. The repulsive force between the protons in the rod

and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod (fig. 1.3a). The region of the sphere nearest the positively charged rod had an excess of negative charges because of the migration of protons away from this location.

If a grounded conducting wire is then connected to the sphere as in (fig. 1.3b) some of the protons leave the sphere and travel to the earth. If the wire to ground is then removed (fig. 1.3c), the conducting sphere is left with an excess of induced negative charge.

Finally, when the rubber rod is removed from the vicinity of the sphere (fig. 1.3d) the negatively induced charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

$$b) \quad k = 9 \times 10^9$$

$$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$$

$$F = 1 \text{ N}$$

$$d = 2 \text{ m}$$

$$F = \frac{kq_1q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1q_2, 5 \times 10^{-5})}{2^2}$$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2^2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2^2$$

$$9 \times 10^9 q_2^2 + 4.5 \times 10^5 q_1 - 4 = 0$$

$$q_1 = 0.0000111 \text{ C}$$

$$q_1 \approx 1.11 \times 10^{-5} \text{ C}$$

$$q_2 = 0.0000388 \text{ C}$$

$$q_2 \approx 3.8 \times 10^{-5} \text{ C}$$

1c. $q_1 = q_2 = 8 \mu\text{C}$

$d = 0.5 \text{ m}$

Determine q if electric field at point P is zero

$\tan \theta = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{0.5}$

$\tan \theta = 2$

$\theta = \tan^{-1}(2)$

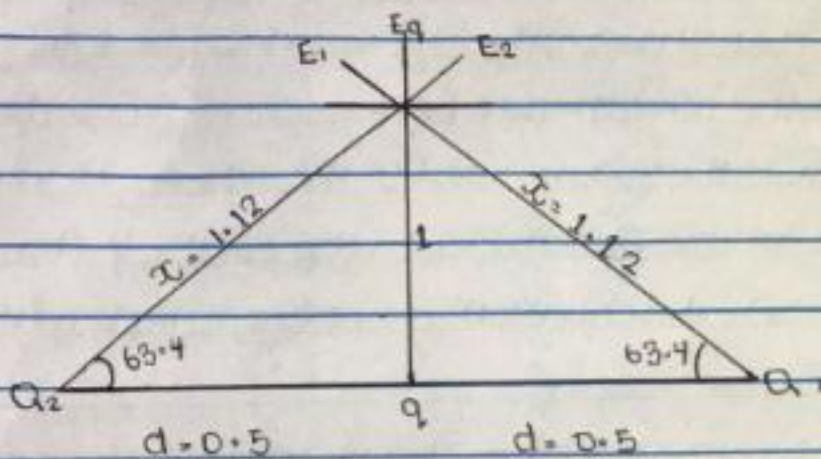
$\theta = 63.4^\circ$

$x^2 = 1^2 + 0.5^2$

$x^2 = 1.25$

$x = \sqrt{1.25}$

$x = 1.12$



$E_1 = E_2$

$E_1 = kq_1 = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{r_e^2} = 5739.795918$

$r_e^2 = 1.12^2$

Magnitude = $\sqrt{(2x)^2 + (2y)^2}$

$E_q = \sqrt{0^2 + 10264.52568^2}$

$0 = 9 \times 10^9 q + 10264.52568$

$q = \frac{-10264.52568}{9 \times 10^9} = -1.140502853 \times 10^{-6}$

$E_q = kq = \frac{9 \times 10^9 \times q}{r^2} = 9 \times 10^9 q$

$r^2 = 1^2$

$q \approx 1.14 \mu\text{C}$

VECTOR

ANGLE

X-COMP

Y-COMP

$E_1 = 5739.795918$

63.4°

$E_1 \cos \theta =$

2570.045785

5132.262839

$E_2 = 5739.795918$

63.4°

2570.045785

5132.262839

$E_q = 9 \times 10^9 q$

90°

$E_q \cos \theta = 0$

10264.52568

~~CORONAVIRUS PANDEMIC & THE EFFECTS OF THE LOCKDOWN~~
~~RESTRICTION OF MOVEMENT ON NIGERIANS~~

a) Volume charge density,

$$\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$$

b) Surface charge density,

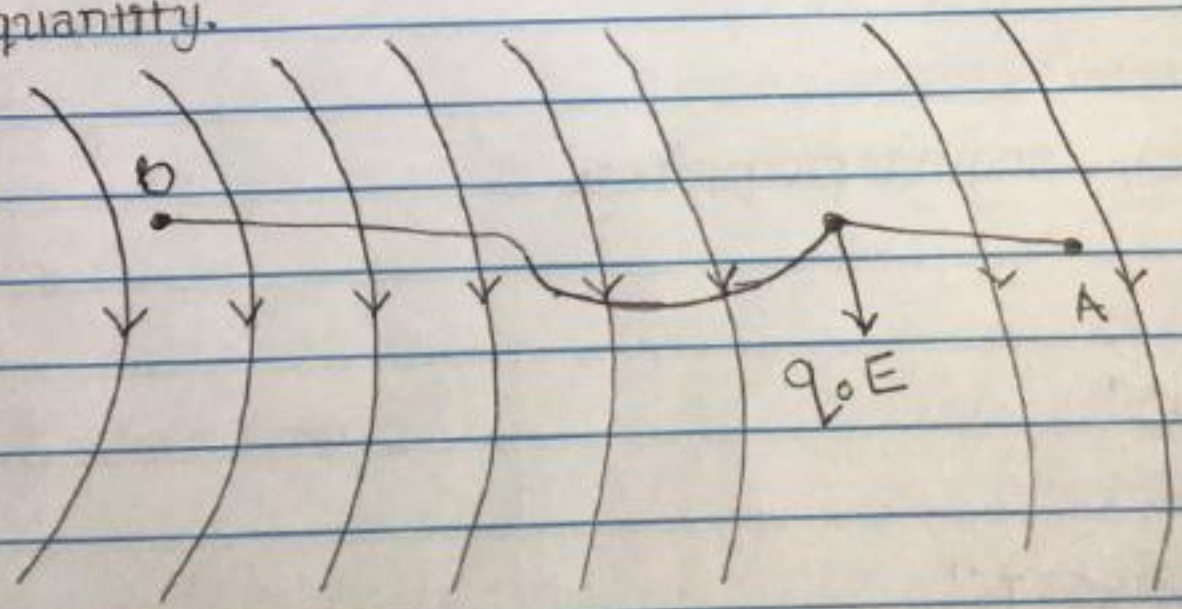
$$\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$$

c) Linear charge density

$$\lambda = \frac{dQ}{dl} \rightarrow dQ = \lambda dl$$

Electrical Potential Difference

The electrical potential difference between two points in an electrical field can be defined as the work done per unit charge ~~between~~ against electrical forces when a charge is transported from one point to another. It is measured in volt (V) or Joules per coulomb (J/C). Electrical potential difference is a scalar quantity.



Consider the diagram above, suppose a test charge, q_0 , is moved from point A to B along an arbitrary path inside in an electric field, E . The electric field exerts a force, $F = q_0 E$ on the charge as shown. To move the charge from A to B

constant velocity, an external force of $F = -q_0 E$ must act on the charge. Therefore the elemental work done, dW , is given as:

$$dW = F \cdot dl \dots (1)$$

$$F = -q_0 E \dots (2)$$

Substituting eq(2) in eq(1) yields

$$dW = -q_0 E dl \dots (3)$$

Then total work done in moving the test charge from A to B is:

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E dl \dots (4)$$

From the definition of electrical potential difference, it follows that:

$$V_B - V_A = \frac{W(A \rightarrow B)_{q_0}}{q_0} \dots (5)$$

Putting eq(4) in (5) yields

$$V_B - V_A = - \int_A^B E dl \dots (6)$$

SECTION B

4a) Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by $\Phi = \int B \cdot dA$

$$4b) m = 9 \times 10^{-31} \text{ Kg}$$

$$r = 1.4 \times 10^{-7} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ weber/meter}^2$$

Cyclotron frequency = angular frequency

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 62222222222.22222 \text{ T}^{-1}$$

Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\theta - \varphi)}{r^2}$$

From diagram,

$$r^2 = x^2 + y^2 \text{ (pythagoras theorem)}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\theta - \varphi)}{x^2 + y^2} \dots (*)$$

$$\text{But } \sin(\theta - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \dots (**)$$

Substituting (**) into (*), we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{1/2} (x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy \dots (***)$$

Using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

Eq (***) becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length, $2a$, of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than x ,

$$(x^2 + a^2)^{1/2} \cong a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

4c. In the question, we were given parameters such as

- i) mass of the electron, 9.11×10^{-31} kg
- ii) A radius of 1.4×10^{-7} m
- iii) magnetic field of 3.5×10^{-1} weber/m²

and we are asked to find the cyclotron frequency which is equal or the same thing as angular speed. It is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall that,

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$\omega = 62222222222.22222 \text{ T}^{-1}$$

Since the cyclotron frequency is equal to angular speed, the cyclotron frequency is $62222222222.22222 \text{ T}^{-1}$, having a ~~freq~~ unit of $1/\text{T}$ which is equal to the unit of frequency.

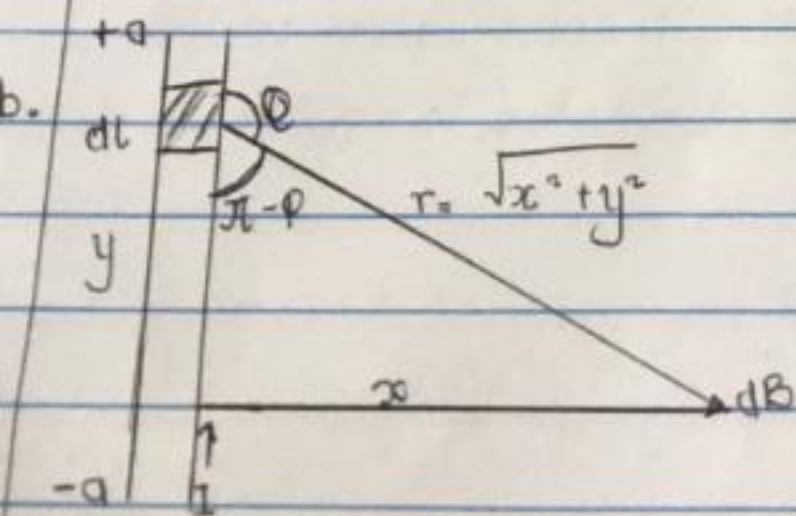
5a Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to the square of the radi

It can be represented mathematically by

$$d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2} \text{ where } \mu_0 \text{ is a constant called permeability of free}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \frac{\text{m}}{\text{A}}$$

The unit of \vec{B} is weber/metre square.



A SECTION OF A STRAIGHT CURRENT CARRYING CONDUCTOR

In a physical situation, we have axial symmetry about the y-axis. Thus, at all points in a circle of radius r , around the conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r} \quad (\#)$$

Equation (#) defines the magnitude of the magnetic field flux density B near a long, straight current carrying conductor.