

(Fig 1.3d) the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

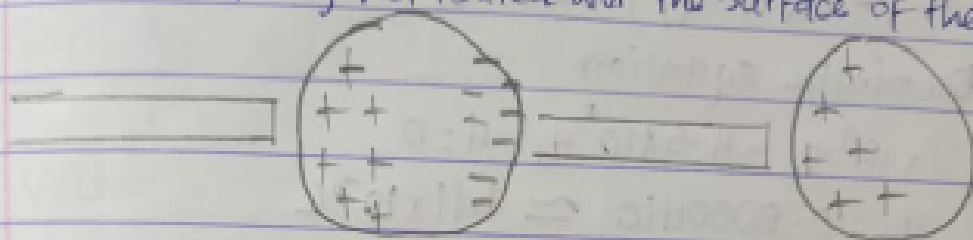


Fig 1.3a \Rightarrow Fig 1.3c

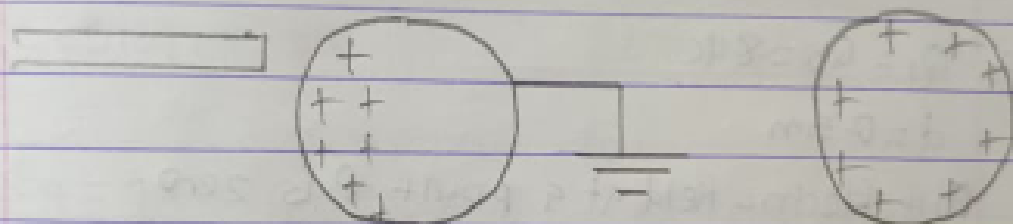


Fig 1.3b

Fig 1.3d

$$16, \quad K = 9 \times 10^9$$

$$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$$

$$F = 1 \text{ N}$$

$$d = 2 \text{ m}$$

Charge on each sphere = ?

$$F = \frac{K q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1 q_2 5 \times 10^{-5})}{2^2}$$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2$$

Quadratic equation

$$9 \times 10^9 q_2 - 4.5 \times 10^5 q_1 + 4 = 0$$

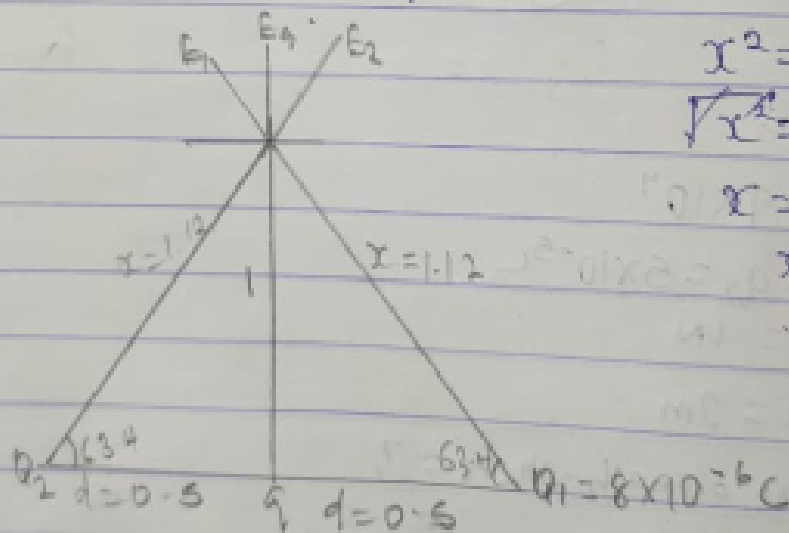
$$q_1 = 0.000011 \text{ C} \approx 1.1 \times 10^{-5} \text{ C}$$

$$q_2 = 0.000038 \text{ C} \approx 3.8 \times 10^{-5} \text{ C}$$

1c. $Q_1 = Q_2 = 8 \mu\text{C}$

$d = 0.5 \text{ m}$

Q. If electric field at a point P is zero.



$$x^2 = 1^2 + 0.5^2$$

$$\sqrt{x^2} = \sqrt{1.25}$$

$$x = \sqrt{1.25}$$

$$x = 1.12 \text{ m}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{1}{0.5}$$

$$\theta = \tan^{-1}(2)$$

$$\theta = 63.4^\circ$$

$$E_1 = K \frac{Q_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 57397.9598$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \text{--- (3)}$$

Using special integrals: $\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$

Equation (3) becomes $B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$

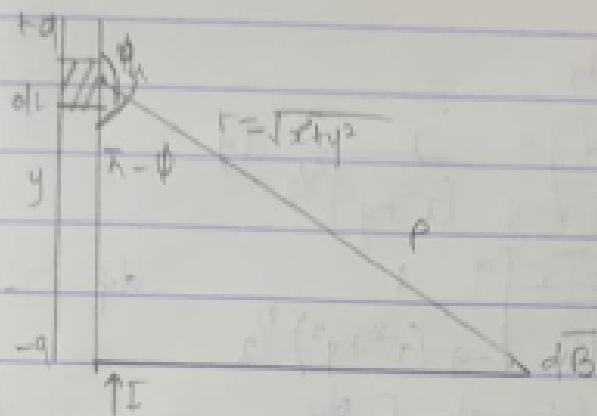
$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than x , $(x^2 + a^2)^{1/2} \approx a$, as $a \rightarrow \infty$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y -axis. Thus at all points in a circle of radius r around the conductor, the magnitude of B is: $B = \frac{\mu_0 I}{2\pi r}$ --- # (magnitude of the magnetic field or flux density B near a long straight current carrying conductor)



A section of a straight current-carrying conductor

Applying the Biot-Savart law, we find the magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \dots \textcircled{1}$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \dots \textcircled{2}$$

$$\text{Substituting } \textcircled{2} \text{ into } \textcircled{1}, B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

3a) Volume charge density, $\rho = \frac{dq}{dV}$ in $dQ = \rho dV$

i) Surface charge density, $\sigma = \frac{dq}{dA}$ in $dQ = \sigma dA$

ii) Linear charge density, $\lambda = \frac{dq}{dl}$ in $dQ = \lambda dl$

3b) Electric potential difference

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to another. It is measured in Volt (V) or Joules per coulomb (J/C). It is a scalar quantity.

Elemental work done dW is given as

$$dW = F \cdot dl \quad \text{--- (1)}$$

But $F = -q_0 E$ --- (2)

Substituting equation (2) in (1) $= dW = -q_0 E dl$ --- (3)

Total work done in moving the test charge from A to B is

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E dl \quad \text{--- (4)}$$

From the definition of electric potential difference, it follows that

$$V_B - V_A = \frac{h}{q_0} (A' \cdot n \cdot B)_{\text{avg}} \quad \text{--- (5)}$$

Putting equation (4) in (5) yields $V_B - V_A = - \int_A^B \mathbf{E} \cdot d\mathbf{l}$ --- (6)

SECTION B

4a Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol Φ . Mathematically given as $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$

4b. $m = 9 \times 10^{-31} \text{ kg}$

$$r = 1.4 \times 10^{-7} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ weber/meter}^2$$

cyclotron frequency = angular speed

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6.22 \times 10^{10} \text{ T}^{-1}$$

4c mass of electron = $9.1 \times 10^{-31} \text{ kg}$

$$F_2 = \frac{Kq_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.2)^2} = 57397.95918$$

$$F_q = \frac{Kq}{r^2} = \frac{9 \times 10^9 \times q}{1} = 9 \times 10^9 q$$

Vector	Angle	X-component	Y-component
$F_1 = 57397.95918$	63.4°	$F_1 \cos \theta =$ 2570.046785	$F_1 \sin \theta =$ 5132.262839
$F_2 = 57397.95918$	63.4°	2570.045785	5132.262839
$F_q = 9 \times 10^9 q$	90°	$F_q \cos \theta = 0$ $F_x = 0$	$9 \times 10^9 q$ $F_y = 10264.52568$

$$\text{Magnitude} = \sqrt{(F_x)^2 + (F_y)^2}$$

$$F_q = \sqrt{(0)^2 + (10264.52568)^2}$$

$$\text{Since } F_q = 0$$

$$0 = 9 \times 10^9 q + 10264.52568$$

Making q subject of formula

$$q = \frac{10264.52568}{9 \times 10^9}$$

$$q = 1.140502853 \times 10^{-6}$$

$$q = 1.14 \mu\text{C}$$