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Matric No.:19/ENG05/061

Date:20/04/2020

Course code:PHY102

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Section A

Explain with the aid of diagram, how you can produce a negatively charged sphere by method of induction.

To produce a negatively charged sphere by induction consider a positively charged rubber rod brought near a neutral uncharged conducting sphere that is insulated so that there is no conducting path to the ground as shown in the diagram below.

Positively charged rod      Neutral sphere

Fig a.                          Fig b.

The repulsive force between the proton in the rod and those in the sphere causes a redistribution of charges on the sphere, so that some protons move to the side of the sphere furthest away from the rod (Fig a). The region of the sphere nearest the positively charged rod has an excess of negative charge because of the migration of protons away from the location. If a grounded conducting wire is then conducted to the sphere as in (Fig b), some of the protons leave the sphere and travel to the earth and if the wire is removed (Fig c) the conducting sphere is left with excess of induced negative charges. Finally when the rubber is removed from the vicinity of the sphere (Fig d) the induced negative charge remains on the ungrounded sphere then becomes evenly distributed around the sphere.

Fig c                          Fig d

b) Each of two small spheres is charged positively, the combined charge being  $5.0 \times 10^{-5} C$ . If each sphere is repelled from each other by a force of 1N when the spheres are 2.0m apart, calculate the charge on each sphere.

SOLUTION

Combined Charge  $q_1 + q_2 = 5.0 \times 10^{-5} C$ ,  $F = 1N$   
 $r = 2.0m$ ,  $K = 9 \times 10^9 Nm^2/C^2$

From Coulomb's law  $F = \frac{Kq_1 q_2}{r^2}$

and Magnetism  
 $q_1 q_2 = 1 \times 2 \times 10^{-5} = 1.44 \times 10^{-10} C^2$   
 $q_1 + q_2 = 5.0 \times 10^{-5} C$        $q_1 q_2 = 4.44 \times 10^{-10} C$  (1)  
 $q_1 = 5.0 \times 10^{-5} - q_2$  in - (2)  
 $= (5.0 \times 10^{-5} - q_2) q_2 = 4.44 \times 10^{-10}$   
 $= q_2^2 + (5.0 \times 10^{-5}) q_2 - 4.44 \times 10^{-10} = 0$   
 Using formula method  
 $q_2 = \frac{(-5.0 \times 10^{-5}) \pm \sqrt{(5.0 \times 10^{-5})^2 - 4 \times (-4.44 \times 10^{-10})}}{2}$

$q_2 = 3.84 \times 10^{-5}$   
 $q_1 = 5.0 \times 10^{-5} - 3.84 \times 10^{-5} = 1.16 \times 10^{-5}$

Three charges were positioned as shown in the figure below. If  $Q_1 = Q_2 = 8\mu C$  and  $d = 0.5m$ , determine  $\theta$  if the electric field at P is zero.

Solution  
 $Q_1 = Q_2 = 8\mu C$   
 To find  $\theta$  use  $\tan \theta = \frac{opp}{adj}$   
 $\tan \theta = \frac{1}{0.5} = 2 \Rightarrow \tan^{-1} 2$   
 $\theta = 63.45^\circ$   
 $Q_2 d = 0.5m$ ,  $d = 0.5m$   
 Pythagoras rule  $c^2 = a^2 + b^2 \therefore x^2 = 1^2 + 0.5^2$   
 $\therefore x = \sqrt{1^2 + 0.5^2} = 1.12m$   
 $E_1 = \frac{Kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = 5739.795918$   
 Since  $q_1 = q_2 \therefore E_2 = 5739.795918$   
 $E_3 = \frac{9 \times 10^9 \times q}{1^2} = 9 \times 10^9 q$   
 vector  $\begin{array}{c|c|c} \theta & -x\text{-component} & y\text{-component} \\ \hline E_1 & 63.4 & 5739.795918 \cos 63.4 & 5739.795918 \sin 63.4 \\ E_2 & 63.4 & -5739.795918 \cos 63.4 & 5739.795918 \sin 63.4 \\ E_3 & 90^\circ & 0 & 9 \times 10^9 q \\ \hline E_x & & 0 & 102672.16 \\ E_y & & & 102672.16 \end{array}$

Magnitude  $= \sqrt{E_x^2 + E_y^2} = \sqrt{0^2 + (102672.16)^2} = 102672.16$

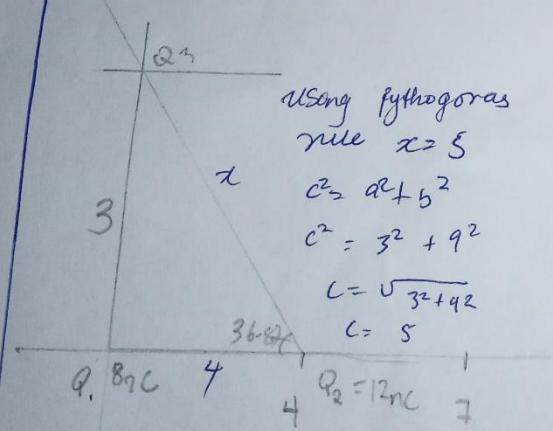
Since  $E_x = 0$   
 $102672.16 - 9 \times 10^9 q = 0$   
 $q = \frac{-102672.16}{9 \times 10^9} = -11.4 \mu C$

- 2) Distinguish between the terms :  
Electric field and electric field  
intensity.

Answer

Electric field is a region of space in which electric charges will experience an electric force while Electric intensity is the force per unit charge.

- b) A positive charge  $Q_1 = 8 \text{ nC}$  is at the origin, and a second positive charge  $Q_2 = 12 \text{ nC}$  is on the  $x$ -axis at  $x = 4 \text{ m}$ . Find i.) The net electric field at a point  $P$  on the  $x$ -axis at  $x = 7 \text{ m}$   
ii.) the electric field at point  $Q$  on the  $y$ -axis at  $y = 3 \text{ m}$  due to charges.



Solution

$$E_1 = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2}$$

$$= 1.469$$

$$E_2 = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2}$$

The net electric field =  $1.469 + 1.469 = 13.5$

$$i) E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj.}} = \frac{3}{4}$$

$$\tan^{-1} \frac{3}{4} = 36.87^\circ$$

Vector	$\theta$	$x$ component	$y$ component
$E_1 = 8 \text{ N/C}$	$90^\circ$	$8 \cos 90^\circ = 0$	$8 \sin 90^\circ = 8$
$E_2 = 4.32 \text{ N/C}$	$36.87^\circ$	$4.32 \cos 36.87^\circ = -3.4584$	$4.32 \sin 36.87^\circ = 2.5920$
		$E_x = -3.4584$	$E_y = 10.5920$

$$\begin{aligned} E_{\text{net}} &= \sqrt{E_x^2 + E_y^2} \\ &= \sqrt{(3.4584)^2 + (10.5920)^2} \\ &= \sqrt{124.08} \\ &= 11.14 \text{ N/C} \end{aligned}$$

**Ques 1(a)** What is magnetic flux? Magnetic flux is defined as the strength of the magnetic field which can be represented by unit of forces. It is represented by the symbol  $\Phi$ , mathematically given as  $\Phi = B \cdot dA$ .

**Ques 1(b)** An electron with a rest mass of  $9.11 \times 10^{-31} \text{ kg}$  moves in a circular orbit of radius  $1.4 \times 10^{-7} \text{ m}$  in a uniform magnetic field of  $3.5 \times 10^{-1} \text{ weber/meter square}$ , perpendicular to the speed with which the electron moves. Find the cyclotron frequency of the moving electron.

Mass,  $m = 9.11 \times 10^{-31} \text{ kg}$   
 radius,  $r = 1.4 \times 10^{-7} \text{ m}$   
 Magnetic field =  $3.5 \times 10^{-1} \text{ weber/m}^2$   
 Cyclotron Frequency = angular speed  
 $w = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{19}}{9.11 \times 10^{-31} \times 1.4 \times 10^{-7}}$   
 $w = 6.222 \times 10^{16} \text{ rad/s}$

**Section B**

It is frequency of an oscillator. Since cyclotron frequency = angular speed it is  $\omega = 6.22222 \text{ rad/s}$ , having unit as  $1/\text{s}$  which is unit of frequency dimensionally.

**Ques 5(a)** State Biot-Savart law. Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space ( $\mu_0$ ), the current ( $I$ ), the change in length, the radius and inversely proportional to square of radius ( $r^2$ ). It can be represented mathematically by =  $\frac{\mu_0 I}{2\pi r}$

**Ques 6** Discuss your answer to 4(b). In 4(b) we were given mass of electron, magnetic field, radius of the electron and we are asked to find cyclotron frequency which is equal to angular speed. It is called cyclotron frequency because

## Magnetic Field of a Straight Current Carrying Conductor

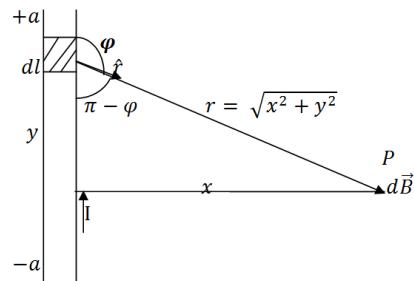


Fig 1: A section of a Straight Current Carrying Conductor

Applying the Biot-Savart law, we find the magnitude of the field  $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin \theta$$

$$\therefore B = \frac{\mu_o I}{4\pi} \int_{-a}^a \frac{dlsin(\pi - \varphi)}{r^2}$$

From diagram,  $r^2 = x^2 + y^2$  (*Pythagoras theorem*)

$$B = \frac{\mu_o I}{4\pi} \int_{-a}^a \frac{dlsin(\pi - \varphi)}{x^2 + y^2} \dots (*)$$

$$\text{But } \sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \dots (**)$$

Substituting  $(**)$  into  $(*)$ , we have

$$B = \frac{\mu_o I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_o I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall  $dl = dy$

$$\begin{aligned} B &= \frac{\mu_o I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy \\ B &= \frac{\mu_o I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \dots (***) \end{aligned}$$

Using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

Equation  $(***)$  therefore becomes

$$\begin{aligned} B &= \frac{\mu_o I x}{4\pi} \left[ \frac{y}{x^2(x^2 + y^2)^{1/2}} \right]_{-a}^a \\ B &= \frac{\mu_o I x}{4\pi} \left( \frac{2a}{x^2(x^2 + a^2)^{1/2}} \right) \\ B &= \frac{\mu_o I}{4\pi x} \left( \frac{2a}{(x^2 + a^2)^{1/2}} \right) \end{aligned}$$

When the length  $2a$  of the conductor is very great in comparison to its distance  $x$  from point P, we consider it infinitely long. That is, when  $a$  is much larger than  $x$ ,

$$(x^2 + a^2)^{1/2} \cong a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_o I}{2\pi x}$$