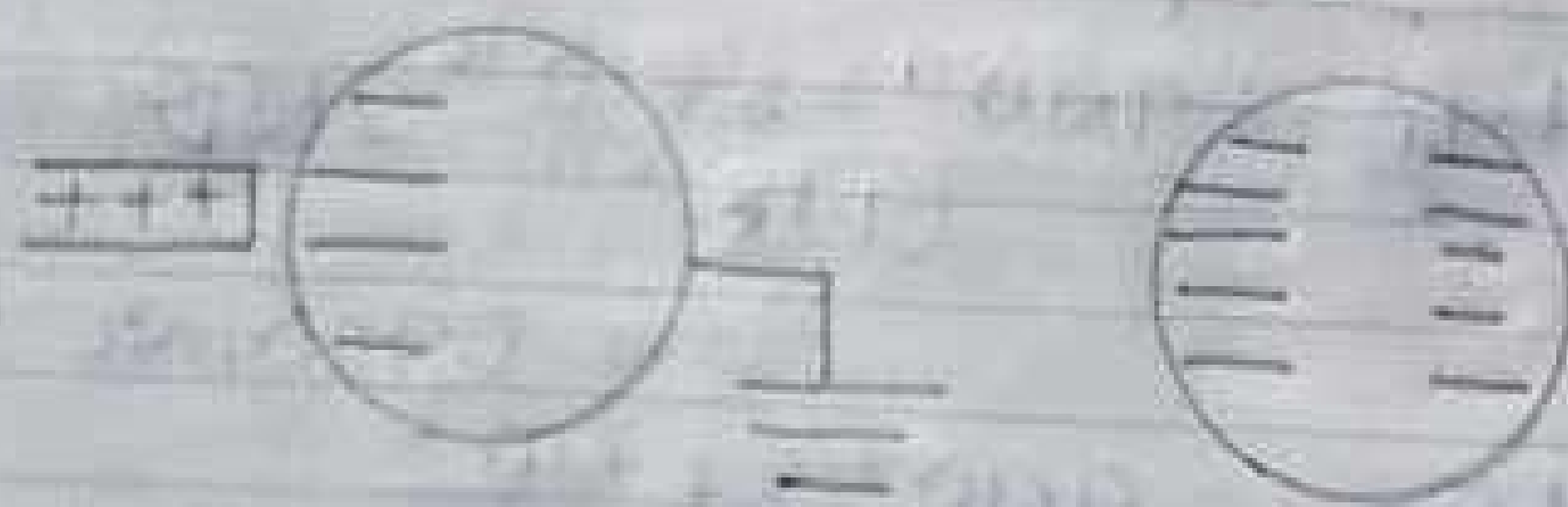
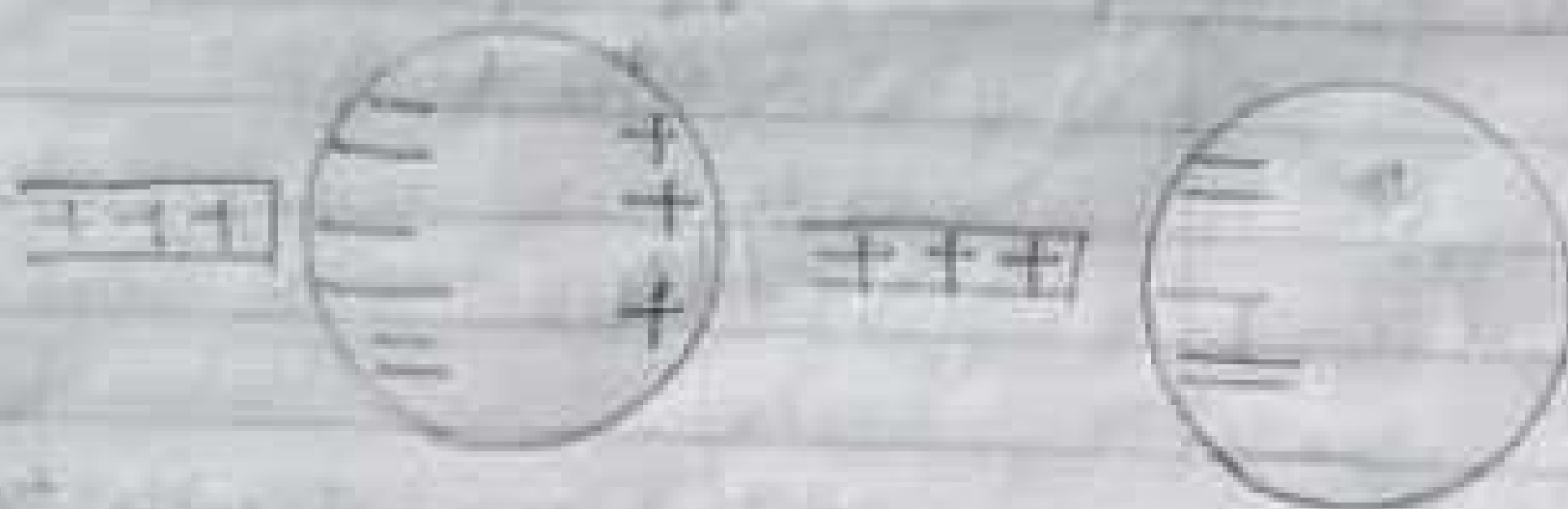


a) Charging by Induction:

Electric charges can be obtained on an object without touch/light, by a process called electrostatic induction.

Consider a positively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conductivity path to ground as shown below. The repulsive force between the protons on the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to the positively charged rod has an excess of negative charge because of the migration of proton away from the location.

Diagram



$$k = 9 \times 10^9$$

$$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$$

$$F = 1 \text{ N}$$

$$d = 2 \text{ m}$$

Calculate the charge on each sphere?

Recall that

$$k = 9 \times 10^9$$

$$F = \frac{kq_1q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times (q_1 q_2 5 \times 10^{-5})}{2^2}$$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^9 q_2$$

It is a quadratic equation

$$9 \times 10^9 q_2 - 4.5 \times 10^5 q_1 + 4 = 0$$

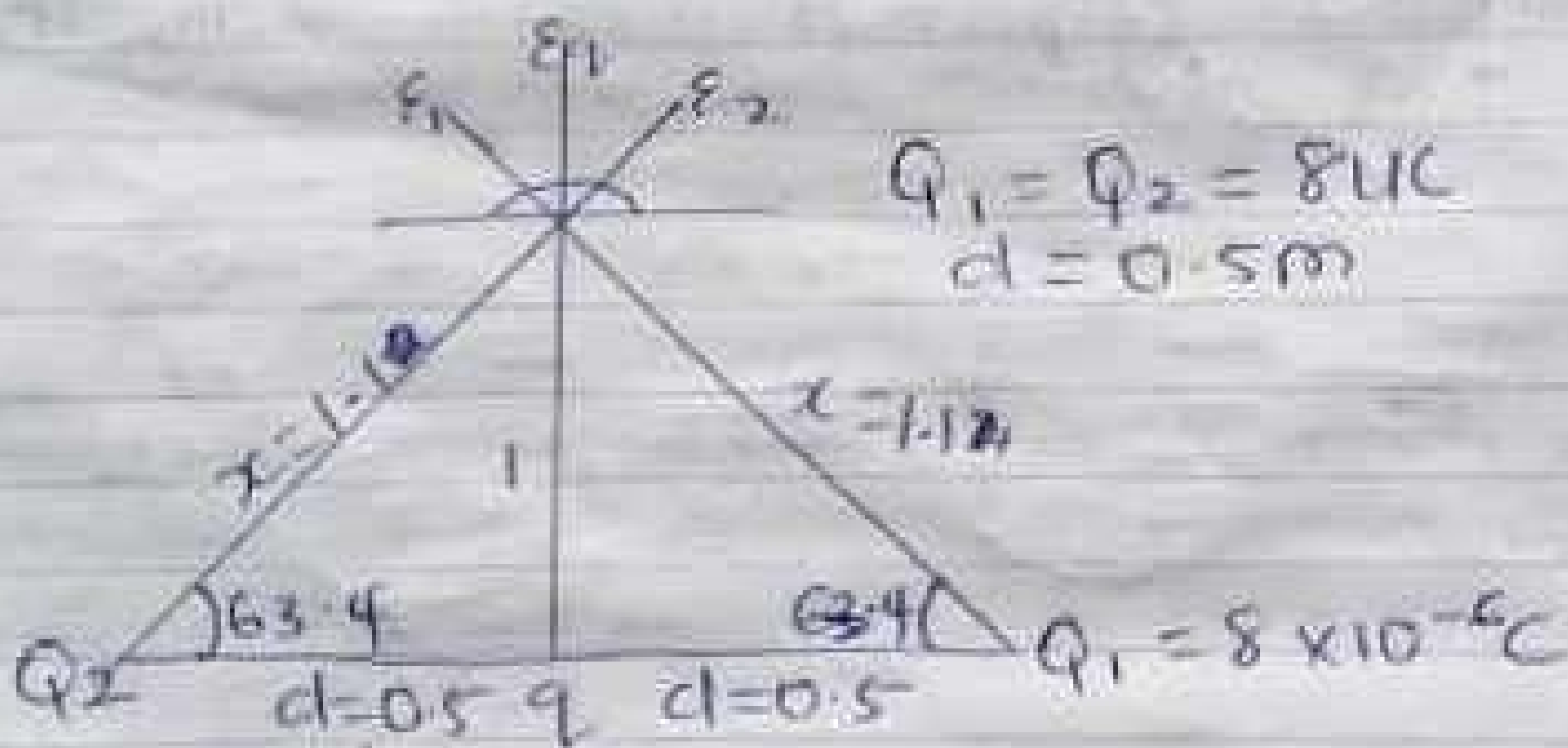
$$q_1 = 0.000011 \text{ C}$$

$$q_2 = 0.000038 \text{ C}$$

$$\underline{q_1} = 1.1 \times 10^{-5} \text{ C}$$

$$\underline{q_2} = 3.8 \times 10^{-5} \text{ C}$$

ii)



$$E_1 = \frac{kq_1}{r_1^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2}$$

$$= 59504 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r_2^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.1)^2} = 59504 \text{ N/C}$$

$$E_{\text{net}} = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 9}{(1)^2} = 9 \times 10^9 \text{ N/C}$$

$$\text{Encl } Q = E_1 + E_2$$

$$E_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8 \text{ N/C}$$

$$E_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32 \text{ N/C}$$

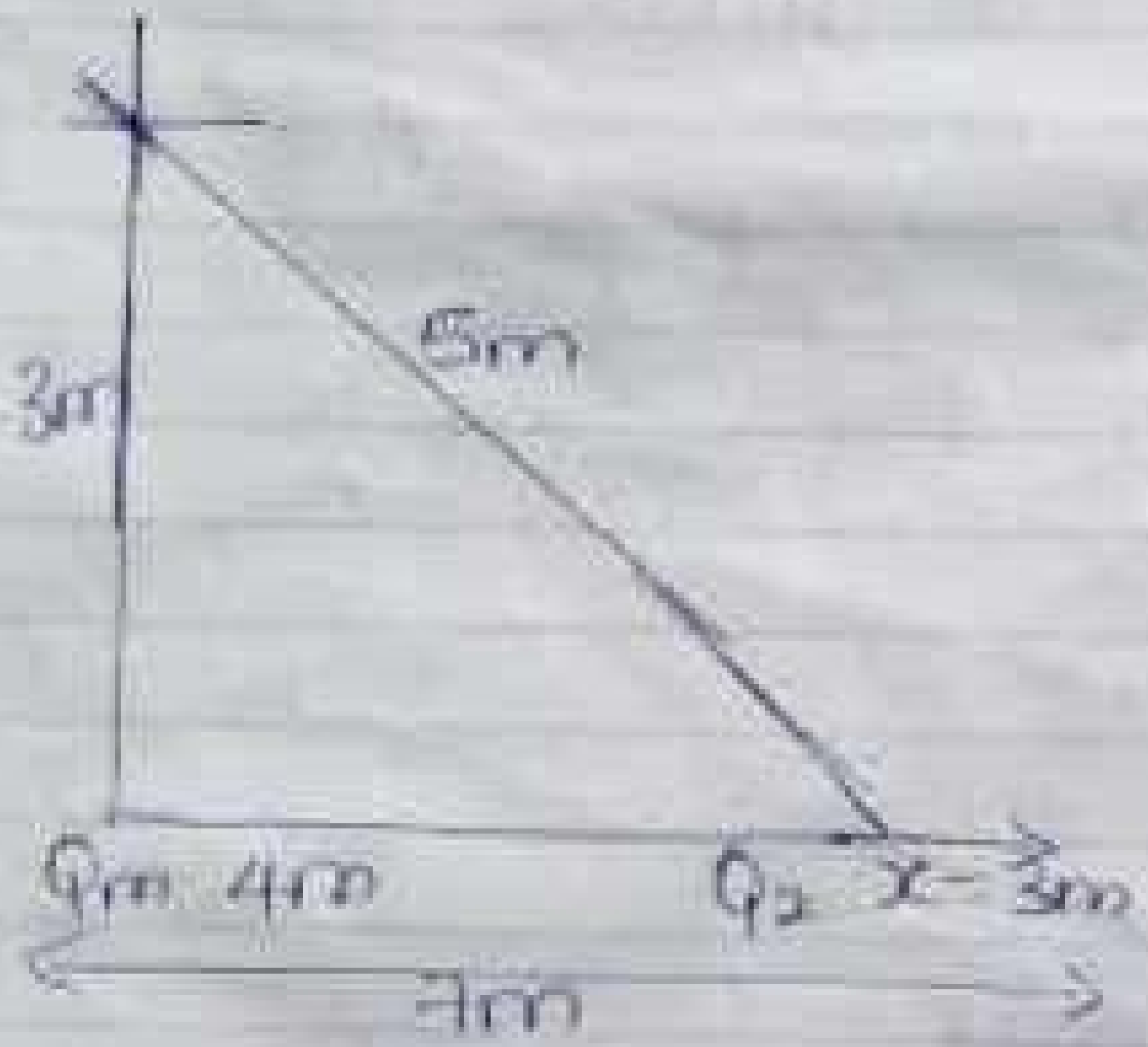
VECTORS	Direction Angle	x-Component	y-Component
$E_1 = 8 \text{ N/C}$	0°	$8 \cos 90 = 0$	$8 \sin 90 = 8$
$E_2 = 4.32 \text{ N/C}$	36.9°	$-4.32 \cos 36.9 = -3.45$ $E_2 x = -3.45$	$4.32 \sin 36.9 = 2.57$ $E_2 y = 2.57$

$$\text{Encl } Q = \sqrt{(-3.45)^2 + (10.57)^2} = 11.14 \text{ N/C}$$

SECTION A

29) An electric field is a region of space in which an electric field charge will experience an electric force while Electric field intensity can be defined as the force per unit charge. Electric field intensity can be expressed mathematically as $E = \frac{F(N)}{q(C)}$

26) $Q_1 = 8nC$
 $Q_2 = 12nC$



1) $E_{net p} = EQ_1 + EQ_2$

$$EQ_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 1.469 N/C$$

$$EQ_2 = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12 N/C$$

$$E_{net p} = 1.469 + 12 = 13.469 \approx 13.5 N/C$$

Vectors	Angle	x-Component	y-Component
$E_1 = 59504$	63.4°	$-59504 \cos 63.4 =$ -26643 N/C $= 26643 \text{ N/C}$	$59504 \sin 63.4 =$ 53205 N/C
$E_2 = 59504$	63.4°	$59504 \cos 63.4 =$ 26643 N/C	$59504 \sin 63.4 =$ 53205 N/C
$E_3 = 9 \times 10^9 \text{ q}$	90°	$9 \times 10^9 \text{ q} \cos 90 = 0$ $\Sigma F_x = 0$	$9 \times 10^9 \text{ q} \sin 90 =$ $9 \times 10^9 \text{ q}$ $\Sigma F_y = 106410 + 9 \times 10^9 \text{ q}$

$$E_p = \sqrt{0^2 + (106410 + 9 \times 10^9 \text{ q})^2}$$

$$E_p = 106410 + 9 \times 10^9 \text{ q}$$

$$106410 + 9 \times 10^9 \text{ q} = 0$$

$$\frac{9 \times 10^9 \text{ q}}{9 \times 10^9} = \frac{-106410}{9 \times 10^9}$$

$$q = -1.182 \times 10^{-5} \text{ C}$$

$$q = -12 \mu\text{C}$$

Section B

4A) Magnetic flux is defined as the strength of magnetic field represented by lines of force. It is represented by the symbol Φ .

Mathematically given as

$$\Phi = B \cdot dA$$

45)

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$r = 1.4 \times 10^{-2} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ weber/meter}^2$$

$$\theta = 90^\circ$$

$$\omega = ?$$

$$q = -1.6 \times 10^{-19} \text{ C}$$

$$\omega = \frac{qB}{m}$$

$$\omega = \frac{-1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$\omega = -0.615 \times 10^{-49}$$

$$\underline{6.15 \times 10^{-50} \text{ rad/s}}$$

46)

In the question we were given some parameters such as

i) mass of the electron = $9.11 \times 10^{-31} \text{ kg}$

ii) magnetic field of $3.5 \times 10^{-1} \text{ weber/meter}^2$

iii) A radius of $1.4 \times 10^{-2} \text{ m}$

And we were asked to find the cyclotron frequency which is equal to the same thing as angular speed. It is called cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall: Angular speed is equal

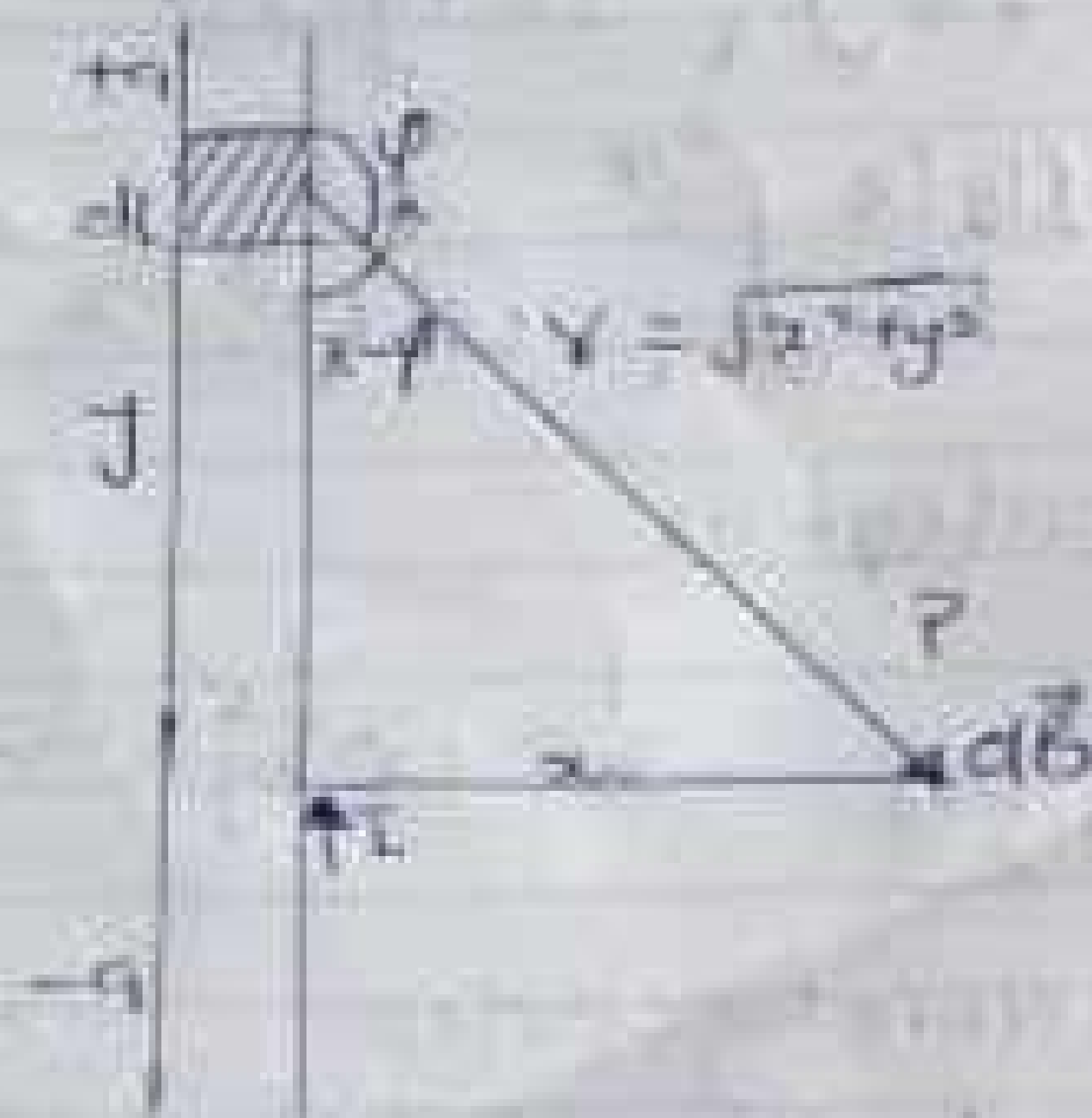
$$\omega = \frac{v}{r} = \frac{qB}{m}$$

5a) Biot Savart law states that the magnetic fields directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to square of radius (r^2). It can be represented mathematically by

$$dB = \frac{\mu_0 I dl \sin \phi}{4\pi r^2}$$

μ_0 is a constant called permeability
 $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$ $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$
 Unit for I is A and for dl is m
 The unit of B is weber/m^2

5b)



Applying Biot Savart law, we find the magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \phi$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dl \sin(\pi - \phi)}{r^2}$$

from diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \text{--- (1)}$$

$$B_{\text{net}}(\vec{x}-\vec{y}) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}} \quad (2)$$

Substituting (2) into (1) we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2+y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2+y^2)^{3/2}} dy \quad (3)$$

Using Special integrals

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation (3) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2+y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length $2a$ of a conductor is great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than

$$(x^2 + a^2)^{1/2} \geq a, \quad a > 0 \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi X}$$

In a physical situation, we have circular symmetry.