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PH1102

1a.) Charging by Induction

Electric charges can be obtained on an object without touching it by a process called Electrostatic Induction

Consider a positively charged rubber rod brought near a neutral uncharged conducting sphere that is insulated so that there is no conducting path to the ground as shown below. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges in the sphere so that some protons move to the sides of the sphere farthest away from the rod.

The region of the sphere nearest the positively charged rod has an excess of negative charges because of the migration of protons away from this location. If a grounded conducting wire is then connected to the sphere as in some of the protons leave the sphere and travel to the earth. If the wire to ground is then removed the conducting sphere is left with an excess of induced negative

Charge

Finally, when the rubber rod is removed from the vicinity of the sphere the negatively induced charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

$$1b.) K = 9 \times 10^9$$

$$q_1 q_2 = 5 \times 10^{-5} \text{ C}$$

$$F = 1 \text{ N}$$

$$d = 2 \text{ m}$$

$$F = \frac{K q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 (q_1 q_2 5 \times 10^{-5})}{2^2}$$

$$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2$$

$$4 = 4.5 \times 10^5 q_1 + 9 \times 10^5 q_2$$

$$9 \times 10^9 q_2 + 4.5 \times 10^5 q_1 = 4$$

$$q_2 = 0.000011 \text{ C}$$

$$q_1 = 1.1 \times 10^{-5} \text{ C}$$

$$q_2 = 0.000038 \text{ C}$$

$$q_2 = 3.8 \times 10^{-5} \text{ C}$$

lc $q_1 = 9.228 \mu\text{C}$
 $d = 0.5 \text{ m}$

Determine the electric field at point P for zero

$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{0.5}$

$\tan \theta = 2$

$\theta = \tan^{-1}(2)$

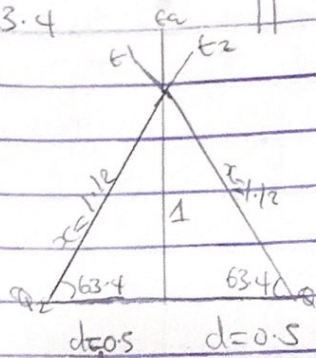
$\theta = 63.4$

$x^2 = 1^2 + 0.5^2$

$x^2 = 1.25$

$x = \sqrt{1.25}$

$x = 1.12$



$E_1 = E_2$

$E_1 = k \frac{q_1}{r^2} = \frac{9 \times 10^9 \times 9.228 \times 10^{-6}}{1.12^2}$

5739.7959

$E_2 = k \frac{q_2}{r^2} = \frac{9 \times 10^9 \times 9.228 \times 10^{-6}}{1^2}$

VECTOR

$E_1 = 5739.795918$

$E_2 = 5739.795918$

$E_3 = 9 \times 10^9 q$

Magnitude = $\sqrt{(2x)^2 + (2y)^2}$

$0_1 = \sqrt{0^2 + 10264.52568^2}$

$0 = 9 \times 10^9 q + 10264.52568$

$q = \frac{10264.52568}{9 \times 10^9} = 1.140028 \times 10^{-6}$

9×10^9

$q = 1.14 \mu\text{C}$

ANGLE

63.4°

63.4°

90°

x-comp

$\cos 63.4 =$

2570.045785

2570.045785

$E_3 \cos \theta = 0$

y-comp

\sin

5132.262839

5132.262839

sin θ

10264.52568

39.) volume charge density

$\rho = \frac{dq}{dV} \rightarrow dq = \rho dV$

II.) Surface charge density

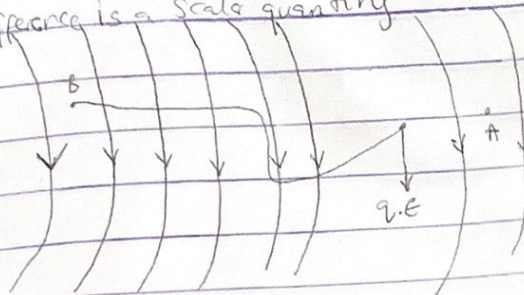
$$\sigma = \frac{dQ}{dA} \rightarrow dA = \frac{dQ}{\sigma}$$

III.) Linear charge density

$$\lambda = \frac{dQ}{dl} \rightarrow dQ = \lambda dl$$

Electrical potential difference

The electrical potential difference between two points in an electrical field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to another. (It is measured in volt (V) or Joules per Coulomb (J/C) electrical potential difference is a scalar quantity.



Consider the diagram, suppose a test charge q is moved from point A to B along an arbitrary path inside an electric field. The electrical field exerts a force, $F = q.E$ on the charge. To move the charge from A to B with constant velocity an external force of $F = -q.E$ must act on the charge. Therefore the elemental work done, dW is given as

$$dW = F \cdot dl \quad \dots (1)$$

$$F = -q.E \quad \dots (2)$$

Substituting eq (2) in eq (1) yields

$$dW = -q.E \cdot dl$$

Then total work done in moving the test charge from A to B is:

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E \cdot dl$$

From the definition of electrical potential difference

$$V_B - V_A = \int_A^B (\mathbf{E} \cdot d\mathbf{l}) \dots (5)$$

Putting eq (5) in (3) yield

$$V_B - V_A = \int_A^B \mathbf{E} \cdot d\mathbf{l} \dots (6)$$

SECTION B

4a Magnetic flux is defined as the strength of the magnetic field which can be represented by line of force. It is represented by $\Phi = \int \mathbf{B} \cdot d\mathbf{A}$

$$(4b) m = 9 \times 10^{-31} \text{ kg}$$

$$r = 1.4 \times 10^{-9} \text{ m}$$

$$B = 3.5 \times 10^{-1} \text{ weber/meter}^2$$

Cyclotron frequency = angular frequency

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$= \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9 \times 10^{-31}}$$

$$\omega = 6222222222.22222 \text{ T}^{-1}$$

4c In the question, we were given parameters such as

i.) mass of the electron = $9.11 \times 10^{-31} \text{ kg}$

ii.) A radius of $1.4 \times 10^{-9} \text{ m}$

iii.) magnetic field of $3.5 \times 10^{-1} \text{ weber/m}^2$

and we are asked to find the cyclotron frequency which is equal to the same thing as angular speed. It is cyclotron frequency because it is a frequency of an accelerator called cyclotron.

Recall that

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

$$\omega = 1.6 \times 10^{-19} \times 3.5 \times 10^{-1}$$

$$9.11 \times 10^{-31}$$

$$\omega = 6222222222.222227^{-1}$$

Since the cyclotron frequency is equal to angular speed, the cyclotron frequency is 6222222222.222227^{-1} , having a unit of $1/s$ which is equal to the unit of frequency.

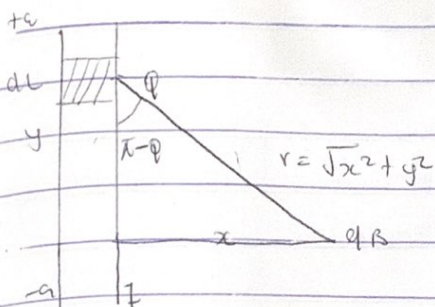
55.) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space \times the current (I) , the change in length, the radius and inversely proportional to the square of the radius. It can be represented mathematically by

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^2}$$

where μ_0 is a constant called permeability

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \frac{\text{m}}{\text{A}}$$

The unit of \vec{B} is $\text{weber/metre square}$



A SECTION OF A STRAIGHT CURRENT CARRYING CONDUCTOR

Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin \phi}{r^2}$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int \frac{\sin(x-\phi) \sin \phi}{r^2}$$

from diagram

$$r^2 = x^2 + y^2 \quad (\text{pythagoras theorem})$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{\sin(\pi - \phi)}{x^2 + y^2} \dots \dots \dots (i)$$

$$\text{But } \sin(\pi - \phi) = \frac{y}{\sqrt{x^2 + y^2}} = \frac{y}{(x^2 + y^2)^{1/2}} \dots \dots (ii)$$

Substituting (ii) into (i)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{y}{(x^2 + y^2)^{1/2}} \frac{y}{(x^2 + y^2)^{1/2}} dy$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{y^2}{(x^2 + y^2)^{3/2}} dy$$

Recall $dy = dx$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy \dots \dots (iii)$$

Using Special Integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \times \frac{y}{(x^2 + y^2)^{1/2}}$$

Eq (iii) becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length, $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is when a is much larger than x

$$(x^2 + a^2)^{1/2} \approx a$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry so $y = \text{same}$. Thus at all points in a circle of radius r surrounding conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r} \quad (iv)$$

Equation (iv) defines the magnitude of the magnetic field flux density B in a long, straight current-carrying conductor.
 magnitude of the magnetic field