

SECTION A.

2a. Electric field is a region of space in which an electric charge will experience an electric force.

Electric field intensity is the force per unit charge. It is measured in N/C.

b. (b) Solution:

$$Q_1 = 8 \text{ nC} = 8 \times 10^{-9} \text{ C}$$

$$Q_2 = 12 \text{ nC} = 12 \times 10^{-9} \text{ C}$$

$$x = 4 \text{ m}$$

(i) Find the electric field at point P at $x = 7 \text{ m}$.

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times (8 \times 10^{-9})}{7^2}$$

$$= \frac{72}{49} = 1.5 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times (12 \times 10^{-9})}{3^2}$$

$$= \frac{108}{9} = 12 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2$$

$$= 1.5 + 12$$

$$= 13.5 \text{ N/C}$$

(ii) At Q,

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times (8 \times 10^{-9})}{3^2} = 8 \text{ N/C}$$

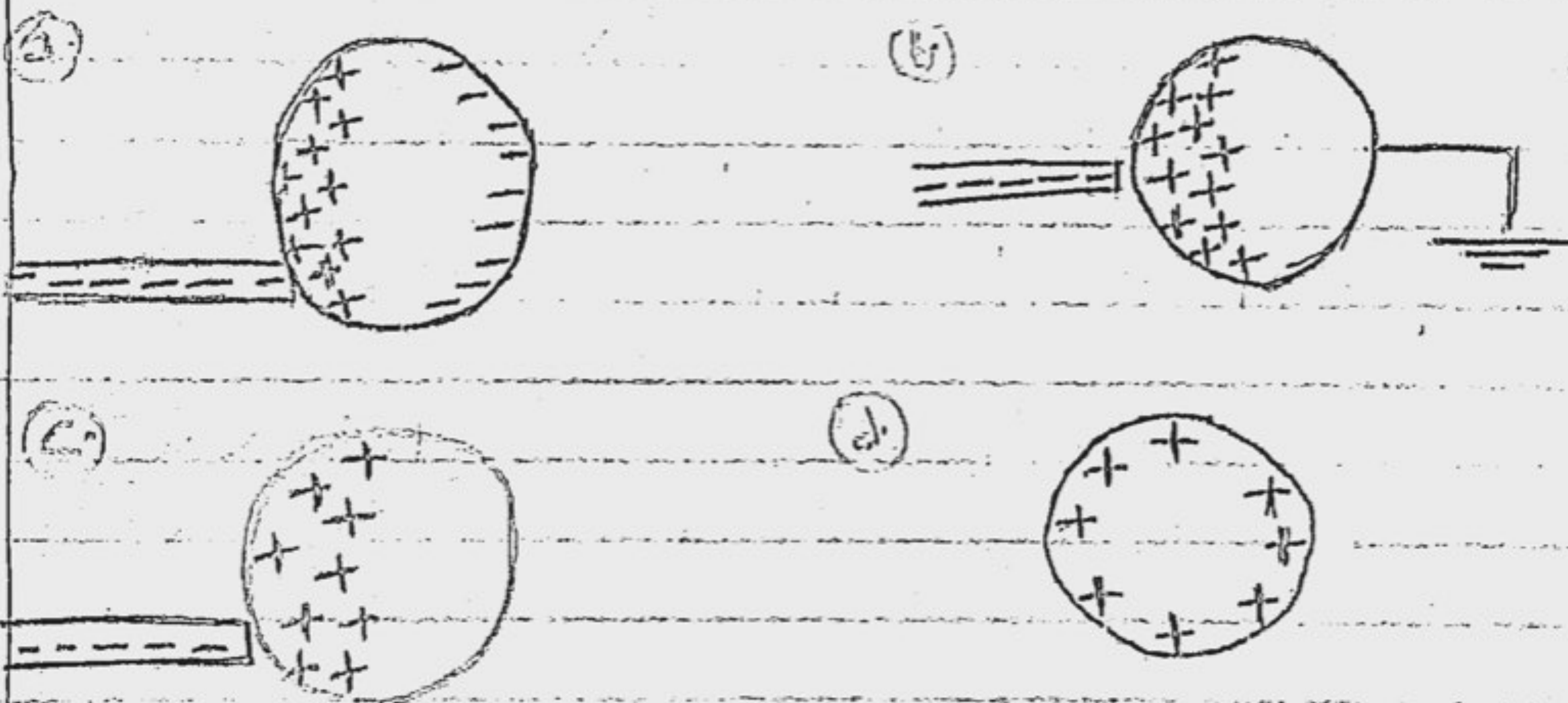
$$E_2 = \frac{kq_2}{r^2} = \frac{(9 \times 10^9) \times (12 \times 10^{-9})}{(5.8)^2} = 3.2 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2$$

$$= 8 + 3.2$$

$$= 11.2 \text{ N/C}$$

1 (a) A neutral conducting sphere, that's insulated, is at rest. A negatively charged rubber rod is brought near the sphere (without touching). The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere. If a grounded conducting wire is then connected to the sphere, some of the electrons leave the sphere and travel to the earth. If the wire to ground is then removed, the conducting sphere is left with an excess of induced positive charge. When the rubber rod is removed from the vicinity of the sphere the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.



(b) Solution,

$$\text{force, } f = 1 \text{ N}$$

$$\text{distance, } r = 2.0 \text{ m}$$

$$q_1 + q_2 = 5.0 \times 10^{-5} \text{ C}$$

$$q_1 = (5.0 \times 10^{-5} - q_2) \text{ C}$$

$$F = \frac{k q_1 q_2}{r^2}$$

$$1 = \frac{(9 \times 10^9) \cdot (5.0 \times 10^{-5} - q_2) q_2}{4}$$

$$4 = (9 \times 10^9) \times (5.0 \times 10^{-5} - q_2) q_2$$

$$4.4 \times 10^{-10} = 5.0 \times 10^{-5} q_2 - q_2^2$$

$$\therefore q_2^2 - 5.0 \times 10^{-5} q_2 + 4.4 \times 10^{-10} = 0$$

$$q_2 = 3.36 \times 10^{-5} \text{ C}, \quad q_1 = 1.64 \times 10^{-5} \text{ C}$$

$$\therefore q_2 = 3.86 \times 10^{-5} \text{ C}$$

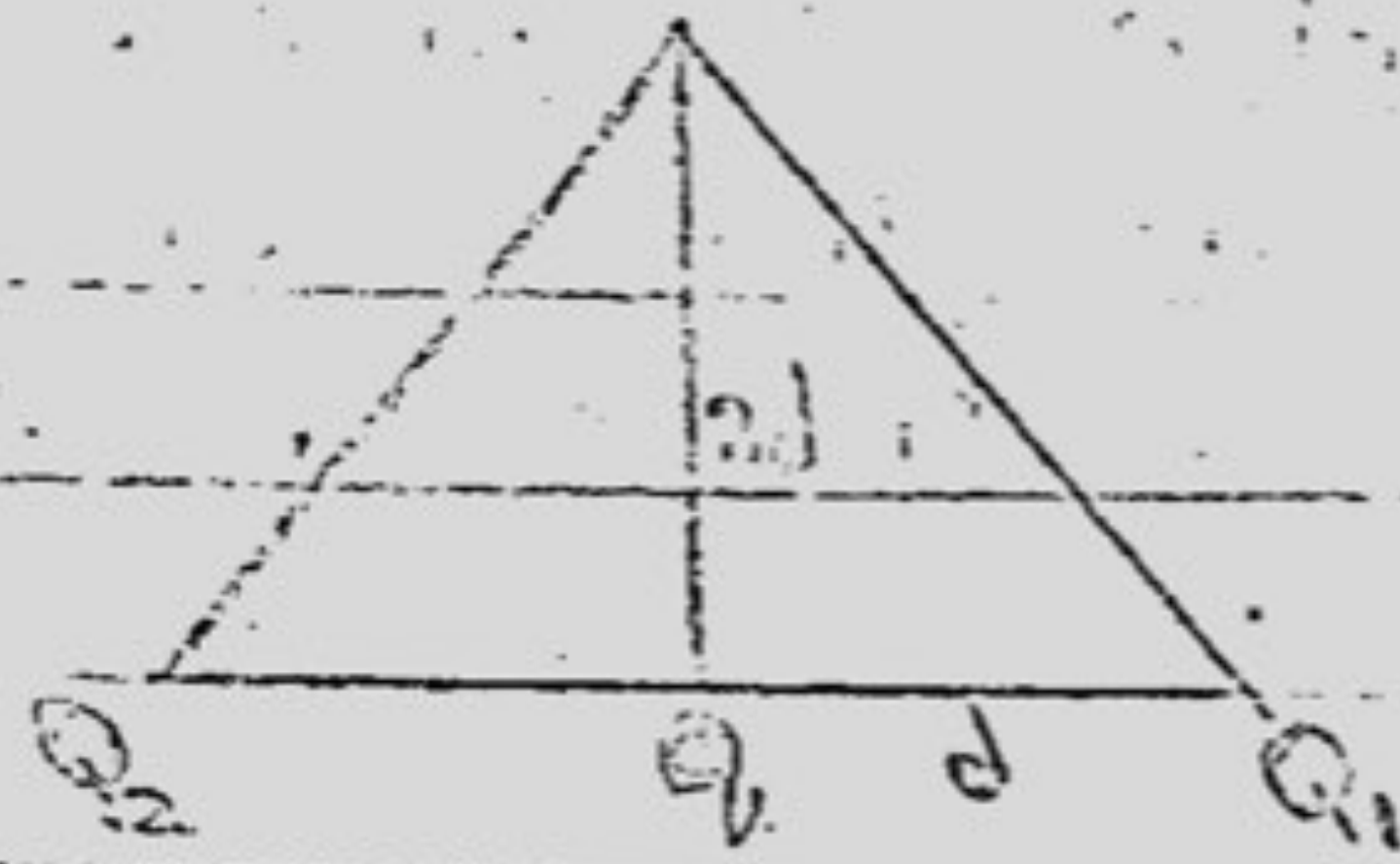
$$q_1 = 1.14 \times 10^{-5} \text{ C}$$

c) solution

$$Q_1 = Q_2 = 8 \text{ nC}$$

$$d = 0.5 \text{ m}$$

$$q = ?$$



$$F = \frac{k Q_1 Q_2}{r^2} = \frac{(9 \times 10^9) \times (8 \times 10^{-6})^2}{r^2}$$

$$= 5.76 \times 10^{-1} \text{ N}$$

When $r = 0.5 \text{ m}$

$$F = 5.76 \times 10^{-1} \text{ N}$$

$$5.76 \times 10^{-1} = \frac{(9 \times 10^9) \times (8 \times 10^{-6}) \cdot q}{0.5^2}$$

$$q = \frac{5.76 \times 10^{-1} \times 0.5^2}{(9 \times 10^9) \times (8 \times 10^{-6})} = \frac{1.44 \times 10^{-1}}{7.2 \times 10^3}$$

$$\therefore q = 2 \times 10^{-5} \text{ C}$$

SECTION B

4a) Magnetic flux is defined as the strength of magnetic field represented by lines of force. It is usually represented by the symbol ϕ .

b) mass, $m = 9.11 \times 10^{-31} \text{ kg}$

radius, $r = 1.4 \times 10^{-7} \text{ m}$

$B = 3.5 \times 10^{-1} \text{ W/m}^2$, $q = 1.6 \times 10^{-19} \text{ C}$

cyclotron frequency, $\omega = ?$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}, \therefore \omega = \frac{qB}{m}$$

$$T = \frac{2 \times \pi \times 9.11 \times 10^{-31}}{(1.6 \times 10^{-19}) \times (3.5 \times 10^{-1})}$$

$$\omega = \frac{(1.6 \times 10^{-19}) \times (3.5 \times 10^{-1})}{9.11 \times 10^{-31}}$$

$$\omega = \frac{5.6 \times 10^{-20}}{9.11 \times 10^{-31}} = 6.15 \times 10^{10} \text{ s}^{-1}$$

The cyclotron frequency of the moving electron circulates in orbit of an angular speed / angular frequency of $6.15 \times 10^{10} \text{ s}^{-1}$

- (5a) The Biot-Savart law is based on the following observations for the magnetic field $d\vec{B}$ at a point ~~pass~~ associated with a length element $d\vec{l}$ of a wire carrying a steady current I .

$$\therefore d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

- (b) Applying Biot-Savart law, we find the magnitude of the field $d\vec{B}$.

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \quad \text{--- (1)}$$

$$\text{But } \sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{--- (2)}$$

Substituting (2) in (1)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \cdot x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \text{--- (3)}$$

using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2 + y^2)^{1/2}}$$

Equation 3 becomes,

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than x ,

$$(x^2 + a^2)^{1/2} \approx a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about their axis. Thus, all points in a circle of radius r , around the conductor, the magnitude B is:

$$B = \frac{\mu_0 I}{2\pi r}$$

This equation defines the magnitude of a magnetic field of flux density B near a long, straight current-carrying conductor.