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$$\int \frac{2x}{\sqrt{4x^2-1}} dx = 2 \int \frac{x}{(4x^2-1)^{1/2}} dx$$

$$u = (4x^2-1)^{1/2}$$

$$u^2 = 4x^2-1 \quad \therefore u^2+1=4x^2$$

$$x = \left\{ \frac{u^2+1}{4} \right\}^{1/2}$$

$$\frac{dx}{du} = \frac{1}{2} \left\{ \frac{u^2+1}{4} \right\}^{-1/2} \cdot \frac{u}{2}$$

$$dx = \frac{u du}{4} \left\{ \frac{u^2+1}{4} \right\}^{-1/2}$$

$$2 \int \left\{ \frac{u^2+1}{4} \right\}^{1/2} \cdot \frac{1}{u} \cdot \frac{u du}{4} \left\{ \frac{u^2+1}{4} \right\}^{-1/2}$$

$$\frac{2}{4} \int \frac{u}{u} du$$

$$\frac{2}{4} \int du = \frac{1}{2} [u] + C$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$

2)

$$\int \frac{\sin^{-1} x}{1-x^2} dx$$

$$= \int \sin^{-1} x (1-x^2)^{-1/2} dx$$

$$\text{Let } u = \sin^{-1} x$$

$$du = (1-x^2)^{-1/2} dx$$

$$\int u du = \frac{u^2}{2} + C$$

$$= \frac{(\sin^{-1} x)^2}{2} + C$$

3)

$$\int (\tan x)^6 \sec^2 x dx$$

$$\text{Let } u = \tan x$$

$$du = \sec^2 x dx$$

we have

$$\int u^6 du = \frac{u^7}{7} + C$$

$$= \frac{(\tan x)^7}{7} + C$$