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DEPT: AERONAUTICAL ENGINEERING
COURSE: ENG 282

ASSIGNMENT

Considering the bacteria population at time intervals of t, $t+9$, $t+18$, $t+27$, $t+36$

$N(t) = N$
$N(t+9) = 3 \times N(t) = 3N$
$N(t+18) = 3 \times N(t+9) = 3 \times 3N = 9N$
$N(t+27) = 3 \times N(t+18) = 3 \times 9N = 27N$
$N(t+36) = 3 \times N(t+27) = 3 \times 27N = 81N$

Also,

$N(t) + \Delta N = N(t+9)$; $\Delta N = 3N - N = 2N$
$N(t+9) + \Delta N = N(t+18)$; $\Delta N = 9N - 3N = 6N$
$N(t+18) + \Delta N = N(t+27)$; $\Delta N = 27N - 9N = 18N$
$N(t+27) + \Delta N = N(t+36)$; $\Delta N = 81N - 27N = 54N$

ΔN has values of $2N, 6N, 18N, 54N, \ldots$ which form a geometric progression

$\therefore \Delta N = ar^{n-1}$
$a = 2N$

for $\Delta N = 6N$, $n = \frac{(t+9) - (t)}{9} + 1 = 2$

$\therefore n = \frac{\Delta t}{9} + 1$

$\therefore$ at $\Delta N = 6N$

$6N = ar^{n-1}$
$6N = 2N \times r^{2-1}$
$\frac{6N}{2N} = r^1$
$r = 3$

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$$\therefore \Delta N = 2N \times 3^{n-1}$$

but $n = \frac{\Delta t}{9} + 1$

$$\therefore \Delta N = 2N \times 3^{\left(\frac{\Delta t}{9} - 1\right) + 1}$$

$$\Delta N = 2N \times 3^{\frac{\Delta t}{9}}$$

$$\frac{\Delta N}{\Delta t} = \frac{2N \times 3^{\frac{\Delta t}{9}}}{\Delta t}$$

$$2\frac{dN}{dt} = \lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{2N \times 3^{(\Delta t/9)}}{\Delta t}$$

By direct substitution, the limit is undefined, Therefore, Applying L'hopital's rule,

$$\lim_{\Delta t \to 0} \frac{2N \times 3^{\left(\frac{\Delta t}{9}\right)}}{\Delta t} = \lim_{\Delta t \to 0} \frac{d\left(2N \times 3^{(\Delta t/9)}\right)/d\Delta t}{\frac{d(\Delta t)}{d(\Delta t)}}$$

$$\frac{d\left(2N \times 3^{\Delta t/9}\right)}{d\Delta t} \quad \text{—} \; ①$$

for $y = na^{kx}$

$$\ln y = \ln\left(na^{kx}\right)$$

~~$\ln y = \ln(k x n x a$~~

$$\ln y = \ln(n) + kx\ln(a)$$

Differentiating implicitly,

$$\frac{dy}{dx}\left(\frac{1}{y}\right) = k\ln(a)$$

$$\frac{dy}{dx} = y \times k\ln a$$

but $y = na^{kx}$

$$\therefore \frac{dy}{dx} = na^{kx} \times k\ln a$$

Applying this principle to the expression $\frac{d\left(2N \times 3^{\frac{\Delta t}{9}}\right)}{d\Delta t}$

$$\frac{d\left(2N \times 3^{\frac{\Delta t}{9}}\right)}{d\Delta t} = 2N \times 3^{\frac{\Delta t}{9}} \times \frac{1}{9} \times \ln 3$$

$$= \frac{2\ln 3 N}{9} \times 3^{\Delta t/9}$$

$$\therefore \lim_{\Delta t \to 0} \frac{2N \times 3^{\frac{\Delta t}{9}}}{\Delta t} = \lim_{\Delta t \to 0} \left(\frac{\frac{2\ln 3 N \times 3^{\Delta t/9}}{9}}{1}\right)$$

By direct substitution

$$\lim_{\Delta t \to 0} \left(\frac{\frac{2\ln 3 N \times 3^{\frac{\Delta t}{9}}}{9}}{1}\right) = \frac{\frac{2 \times \ln 3 \times N \times 3^{\left(0/9\right)}}{9}}{1}$$

$$= \frac{2\ln 3 \times N}{9}$$

$$\therefore 2\frac{dN}{dt} = \frac{2\ln 3 \times N}{9}$$

$$\frac{dN}{dt} = \frac{\ln 3 \times N}{9}$$

$$\frac{dN}{N} = \frac{\ln 3}{9} dt$$

$$\int \frac{dN}{N} = \int \frac{\ln 3}{9} dt$$

$$\ln N = \frac{\ln 3 \times t}{9} + k$$

$$N = e^{\left(\frac{\ln 3}{9} \times t + k\right)}$$

$$N = e^{k} \times e^{\frac{\ln 3}{9} \times t}$$

$$N = e^{k} \times \left(e^{\ln 3}\right)^{(t/9)}$$

$$N = e^{k} \times 3^{t/9}$$

let $e^{k} = N_0$,

$\cancel{N = N_0 e}$

$N = N_0 \times 3^{t/9}$

for Sample A, at $t=0$, $N=50$

$\therefore 50 = N_0 \times 3^{0/9}$

$50 = N_0$

$\therefore N = N_0 \times 3^{t/9}$

$N = 50 \times 3^{t/9}$

$N \to f(t) = 50 \times 3^{t/9}$ (sample A)

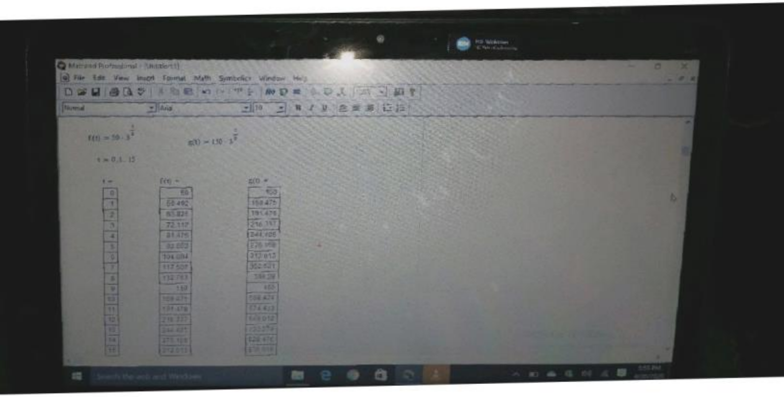
for Sample B, at $t=0$, $N=150$

$150 = N_0 \times 3^{0/9}$

$150 = N_0$

$\therefore N = 150 \times 3^{t/9}$

$N \to g(t) = 150 \times 3^{t/9}$ (sample B)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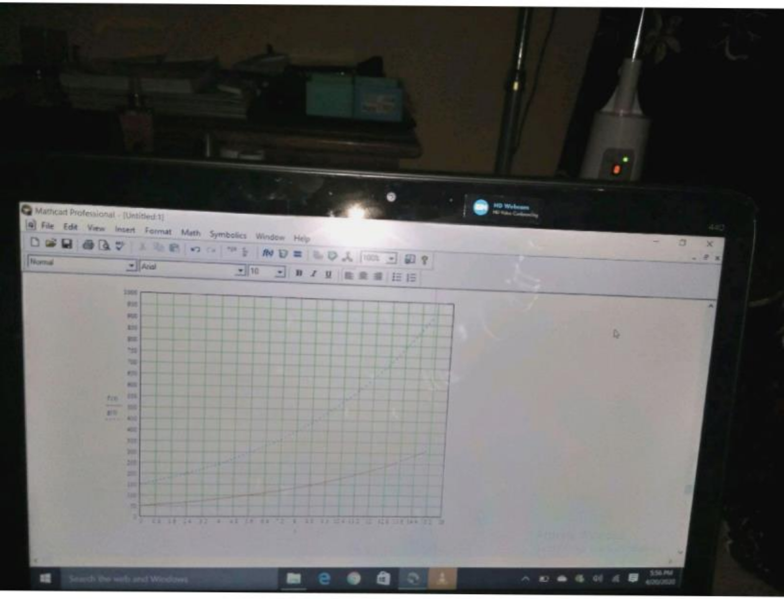
f(t) =
g(t) =



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