

18/SCI01/068

$$1) \quad \mathbf{A} = \begin{bmatrix} 1 & 4 & 8 \\ -3 & 0 & 5 \\ 6 & 2 & 1 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 & 6 & 1 \\ 4 & -7 & -3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$i) \quad |\mathbf{A}| = 1(0-10) - 4(-3-30) + 8(-6-0) \\ = 74 \neq 0, \text{ hence rank} = 3$$

$$ii) \quad \mathbf{B}^T = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{bmatrix}$$

$$|\mathbf{B}^T| = 1(8-3) - 1(-4-0) + 2(3-0) \\ = 15 \neq 0, \text{ hence rank} = 3$$

$$iii) \quad \mathbf{A} + \mathbf{C} = \begin{bmatrix} 1 & 10 & 9 \\ 1 & -7 & 2 \\ 9 & 3 & 3 \end{bmatrix} \quad (\mathbf{A} + \mathbf{C})^T = \begin{bmatrix} 1 & 1 & 9 \\ 10 & -7 & 3 \\ 9 & 2 & 3 \end{bmatrix}$$

$$|(\mathbf{A} + \mathbf{C})^T| = 1(-21-6) - 1(30-27) + 9(20+63) \\ = 717 \neq 0, \text{ hence rank} = 3$$

$$iv) \quad \mathbf{B} + \mathbf{C} = \begin{bmatrix} 1 & 7 & 1 \\ 5 & -9 & 0 \\ 5 & 2 & -2 \end{bmatrix}$$

$$|\mathbf{B} + \mathbf{C}| = 1(18-0) - 7(-10-0) + 1(10+45) \\ = 143 \neq 0, \text{ hence rank} = 3$$

$$v) \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = \begin{bmatrix} 2 & 11 & 9 \\ 2 & -9 & 5 \\ 11 & 4 & -1 \end{bmatrix}$$

$$|\mathbf{A} + \mathbf{B} + \mathbf{C}| = 2(9-20) - 11(-2-55) + 9(8+99)$$

= 1568  $\neq$  0, hence rank = 3