0-4-2020 ANALOG MULTIPLIERS.

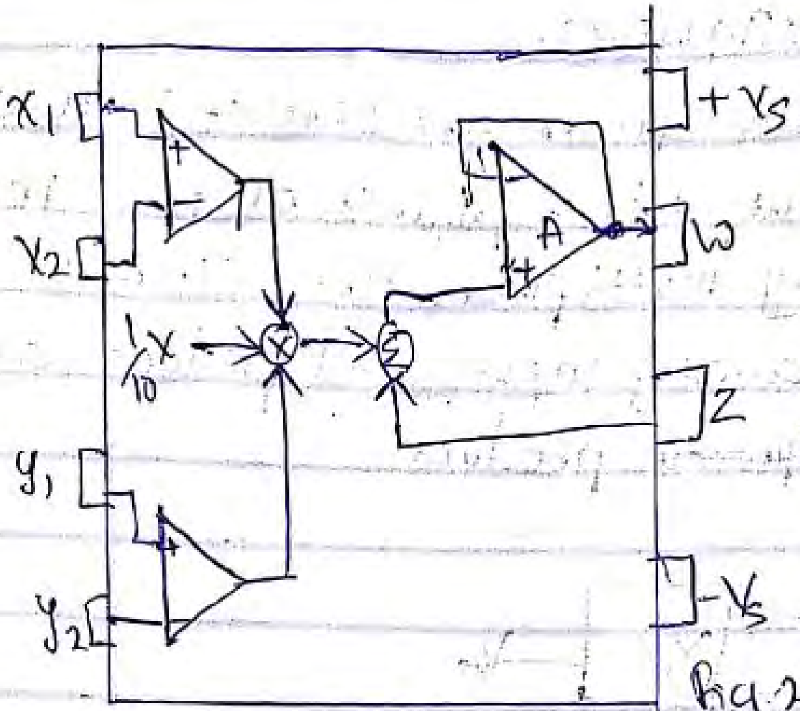
Analog multipliers ~~are~~ circuit is a circuit where the output is a product of two input signal. Analog multipliers are circuits which takes two analog inputs and produce a proportional ~~produ~~ product

V.

$V_o = K \cdot V_x V_y$ K = scaling ~~factor~~

Type	Vx	Vy	Vo
Single quadrant	Unipolar	unipolar	Unipolar
Two quadrant	Bipolar	Unipolar	Bipolar
four quadrant	Bipolar	Bipolar	Bipolar

Analog multipliers can be implemented as IC or as discrete circuits.



$$V_o = \frac{(x_1 - x_2)(y_1 - y_2)}{10} + Z$$

Characteristics
four quadrant
$f_s = 10x$

Fig 2: AD 633

Important Characterisics

(1) Accuracy

(i) Total error ~~SET~~ — % fs

(ii) DC offset — (V_o for $x - y = \phi$)

(iii) Non-linearity — (maximum deviation from ideal when DC error nulled)

(iv) x, y feedthrough (V_o when either x or y are zero)

(2) Dynamic performance

(i) Small signal bandwidth (f such that $|V_o| = \frac{1}{\sqrt{2}}|V_o|_{max}$)

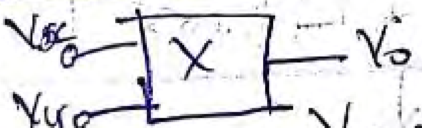
(ii) Settling time (time in which it changes from input to $V_o = V_o \pm \Delta V_o$)

(iv) Slew rate ($\frac{dV_{out}}{dt}$ (max) ⇒ Full power bandwith

Applications of analog multipliers

1. ~~Math~~ mathematical operations ($\times$, $\div$, $\wedge 2$, Rms)
2. frequency doubling & Shifting
3. modulators & demodulators

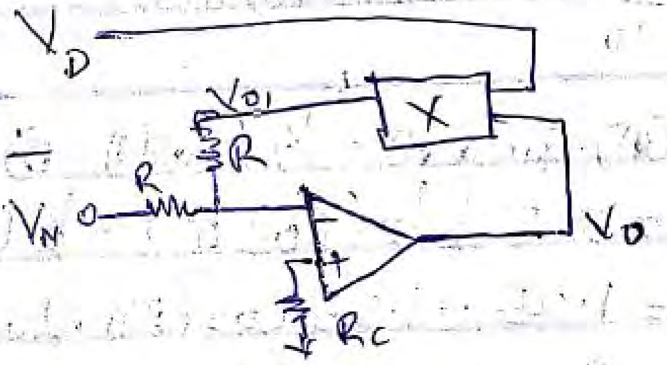
Mathematical operations

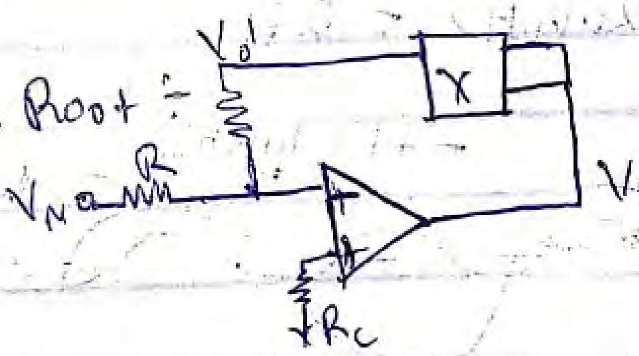
1. multiplication :- 

$V_o = K \cdot V_x \cdot V_y$

2. Squaring :- 

$V_o = K \cdot V_x^2$

3. Division :- 

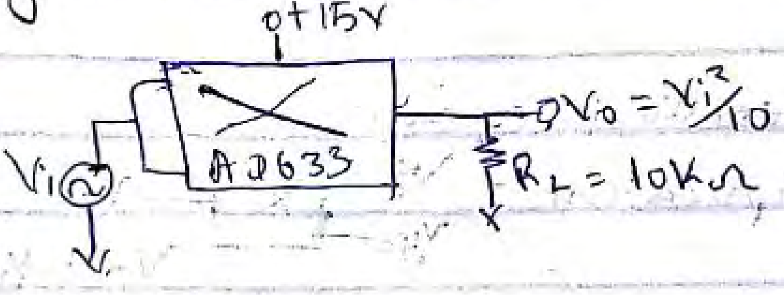
4. Square Root :- 

Design Consideration for Analog multiplier

1. op-amp must be in the linear Region.
2. your feedback must be negative.
3.

Frequency - doubling / Shifting.

The frequency doubler circuit provides an output signal equals to twice the input signal of the circuit.



$V_i = 5 \cdot \sin(2\pi\, 10{,}000t)$

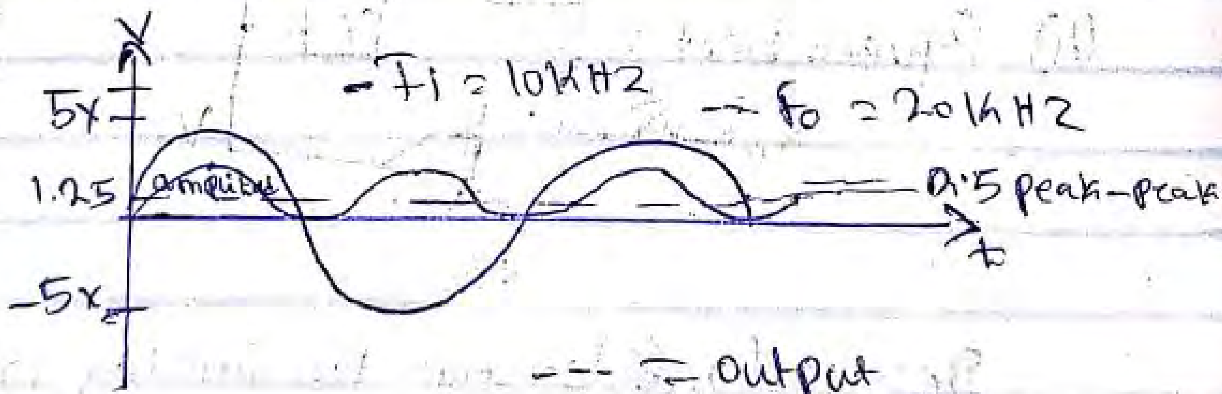
$V_o = \frac{5^2}{10} (\sin(2\pi 10{,}000t)^2$

$\sin(A) \cdot \sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A\ \ B)]$

$V_o = 2.5 \left[\frac{1}{2}(\cos 0^\circ) - \frac{1}{2}\cos(2\pi \cdot 20{,}000t)\right)$

$V_o = 1.25 - 1.25 \cos(2\pi \cdot 20{,}000t)$

$F = 20KHz = 2f_i$



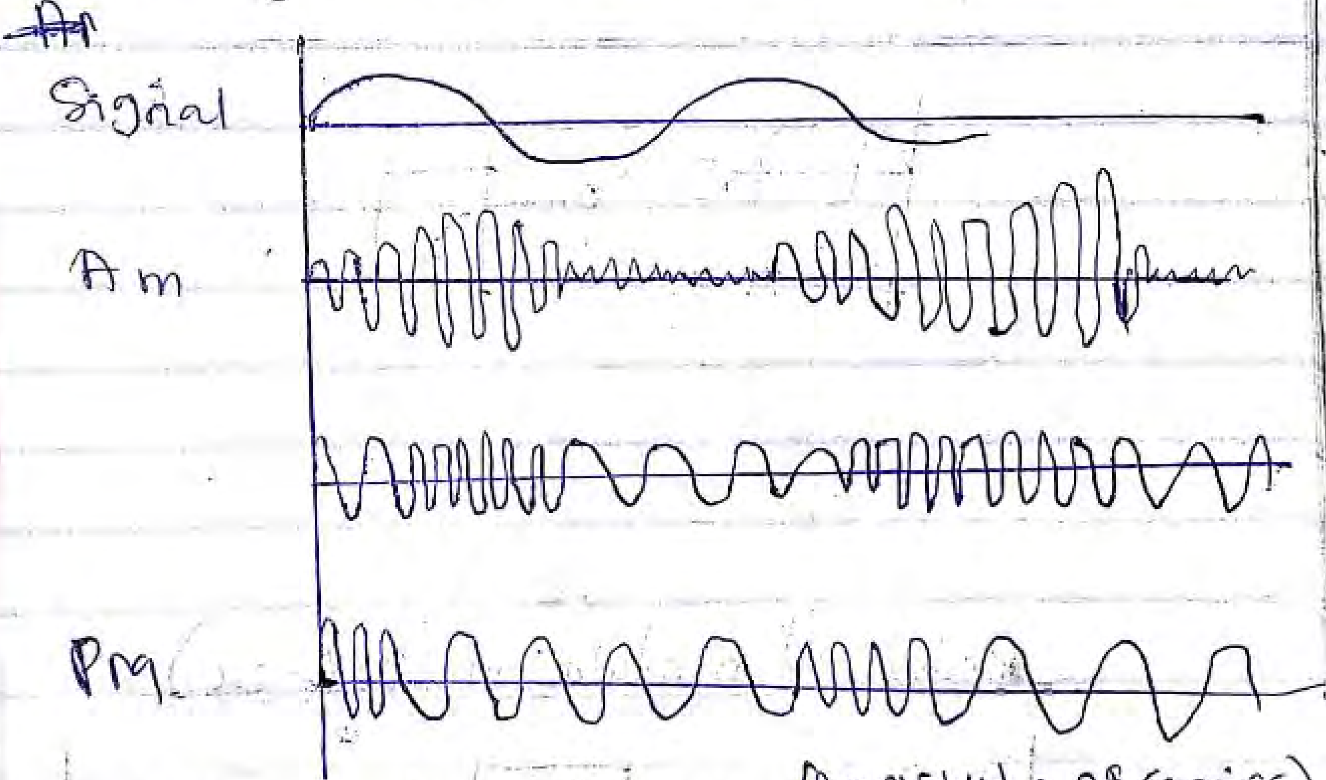
--- = output

— = input.

Signal modulation & demodulation.

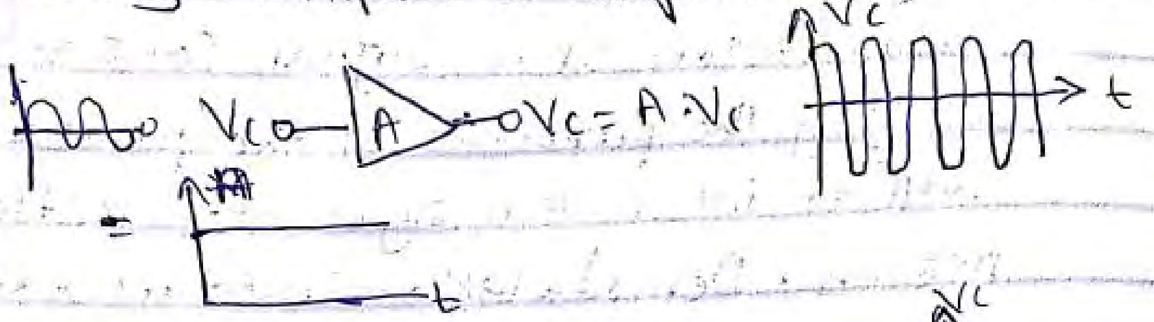
Modulation of a signal consist of varying the properties of the periodic signals with an information signal for better transmission. Demodulation involves extracting original signal from modulated signal.

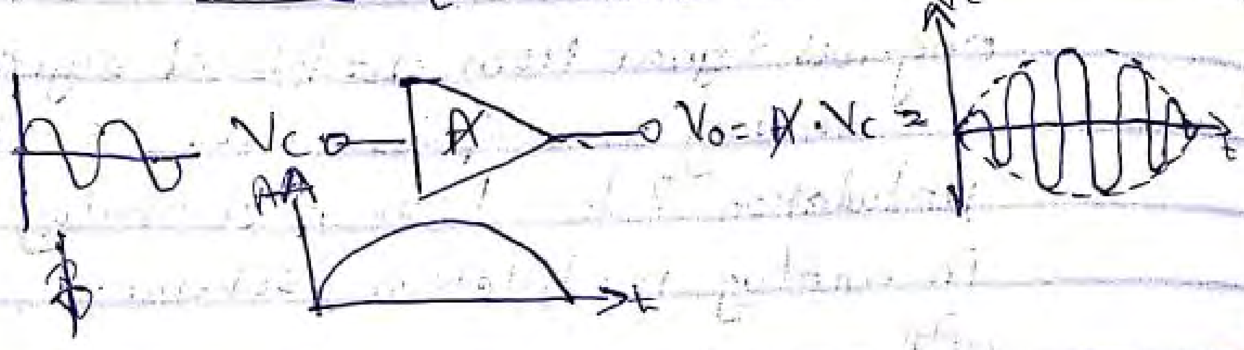
Analog multipliers are usin in modulation & demodulation of analog signal use in analog modulation scheme.

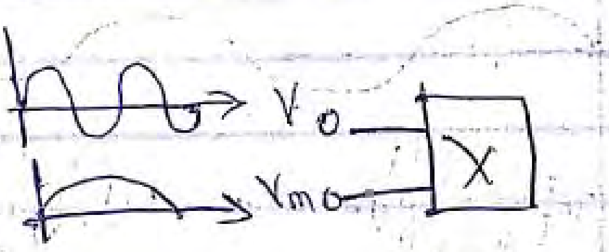


Am :- Amplitude modulation (Amplitude of carrier ∝ modulating signal)
Fm :- Frequency modulation (freq of carrier ∝ modulating signal)
Pm :- Phase modulation (phase of carrier ∝ modulating signal)

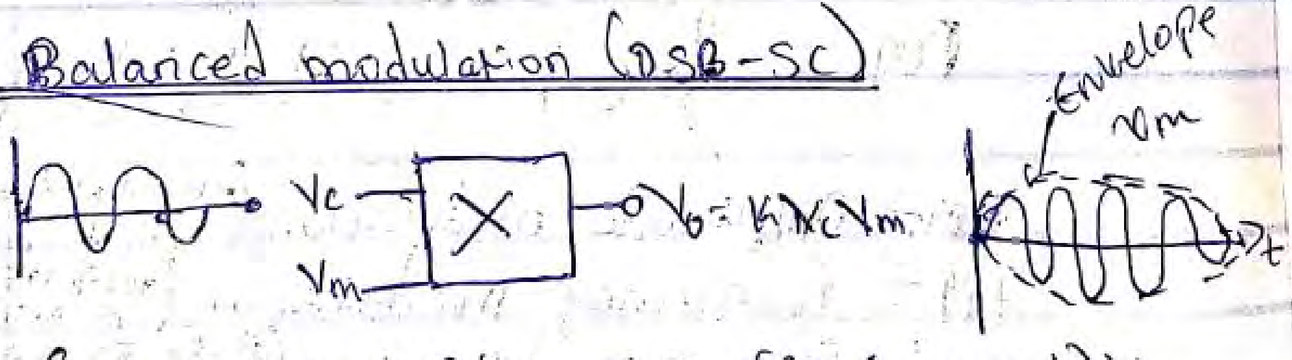
Analog multiplier in Amplitude modulation







Balanced modulation (DSB-SC)



Carrier signal: $V_c = 5 \sin(2\pi \cdot 10{,}000\, t)\, V$

modulating signal $V_m = 5 \sin(2\pi \cdot 6000\, t)\, V$

~~DSB (Double side-band)~~

~~$V_0 = \frac{25}{10} \sin(2\pi 10{,}000 t) \sin(2\pi 1000 t)$~~

Carrier signal: $V_c = V_{cp} \cdot \sin(2\pi f_c t)$

Modulating Signal: $V_m = V_{mp} \cdot \sin(2\pi f_m t)$

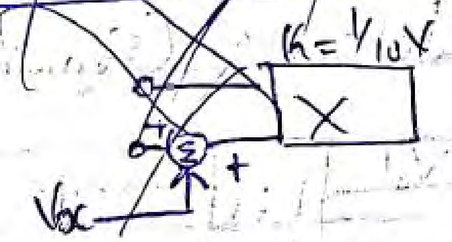
$V_0 = K \cdot V_c V_m$

$= K \cdot V_{cp} \cdot V_{mp} \sin(2\pi F_c t) \sin(2\pi F_m t)$

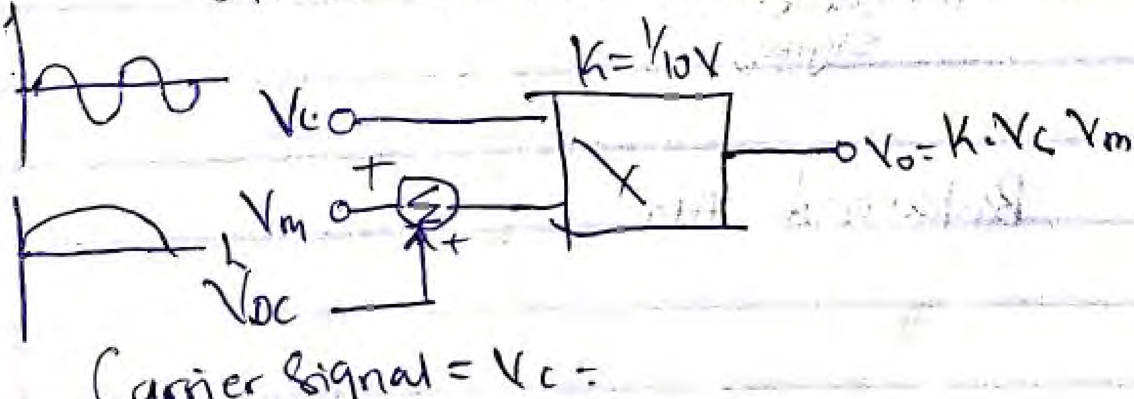
$\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$V_0 = \frac{K}{2} V_{cp} \cdot V_{mp} [\cos(2\pi(F_c - F_m)t) - \cos(2\pi(F_c + F_m)t)]$

~~Standard modulation~~



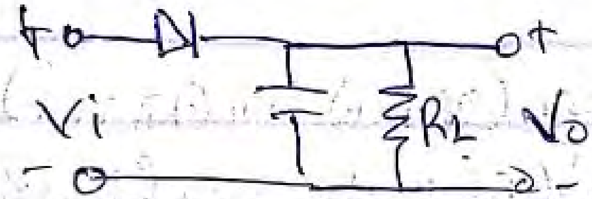
Standard Am modulation Scheme.



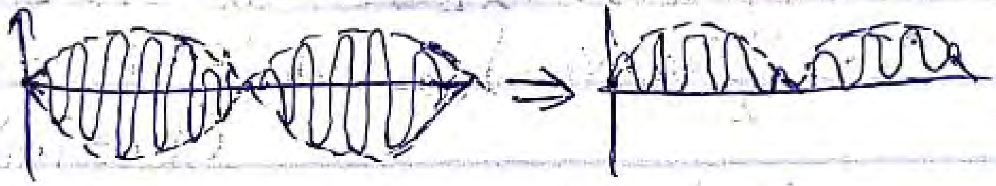
Carrier Signal = V_c =

AM Demodulator: [Demodulation / deductor]

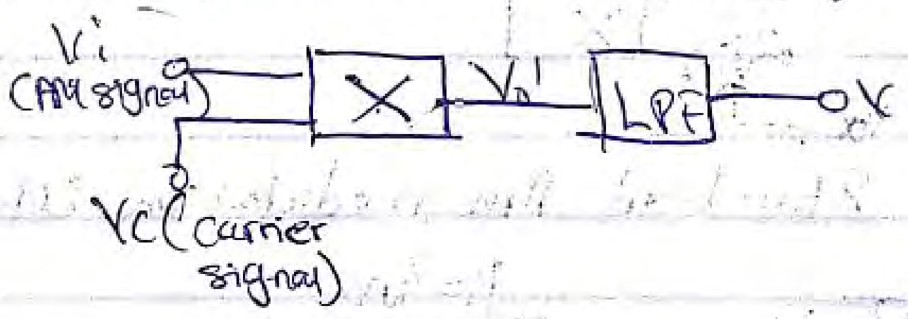
- AM demodulator receives modulated signal & extracts information signal
- Examples are
- Diode AM detector (asynchronous dector) :-



[produces half wave rectification, and modulates in amplitudes.



2.) Multiplier AM detector (Synchronous)



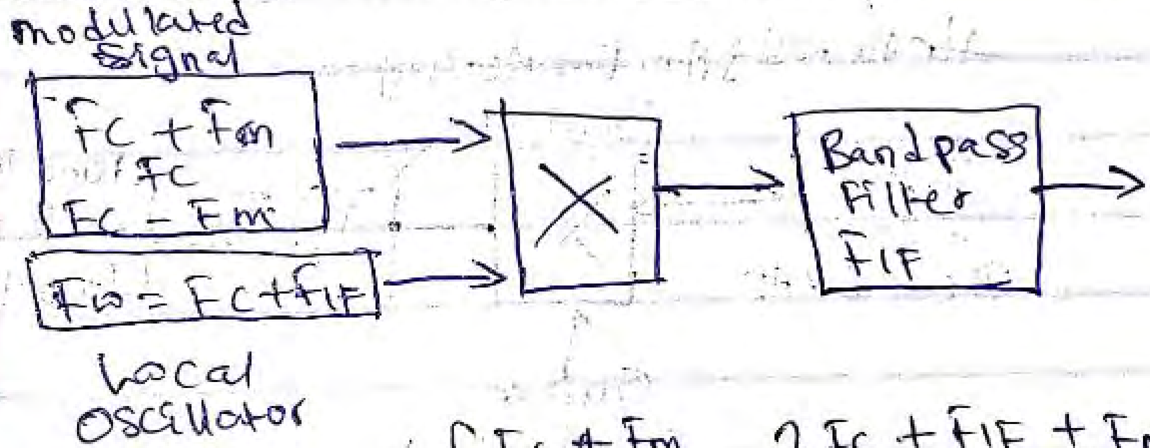
Balanced AM

Radio-Frequency Signal

An RF signal shifts a (modulated) RF signal to an intermediate frequency (IF) prior to demodulation.

- Audio Receivers usually have narrow band tuned high-gain IF amplifiers. It is easier to design for a given IF frequency & shift any incoming signals to that frequency. it is programmable.

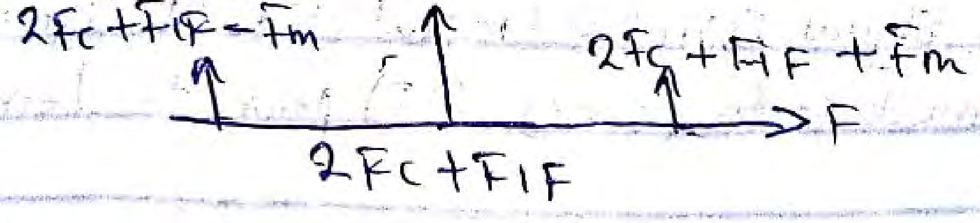
- frequency shifting ≡ Heterodyning ⟹ Heterodyne Receiver.



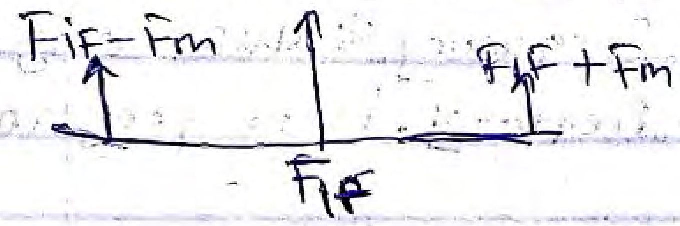
$$\text{Sum: } F_c + F_{IF} + \begin{cases} F_c + F_m = 2F_c + F_{IF} + F_m \\ F_c \quad = 2F_c + F_{IF} \\ F_c - F_m = 2F_c + F_{IF} - F_m \end{cases}$$

$$\text{difference: } F_c + F_{IF} - \begin{cases} F_c + F_m = F_{IF} - F_m \\ F_c \quad = F_{IF} \\ F_c - F_m = F_{IF} + F_m \end{cases}$$

For the Sum

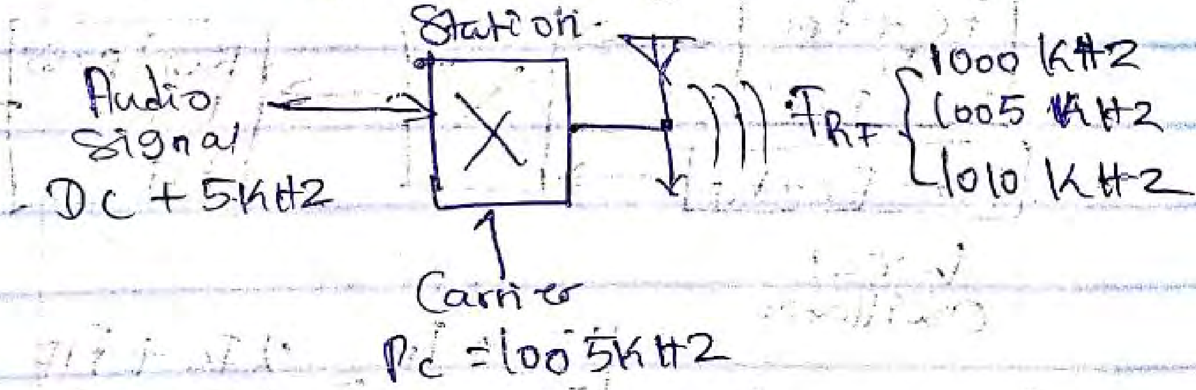


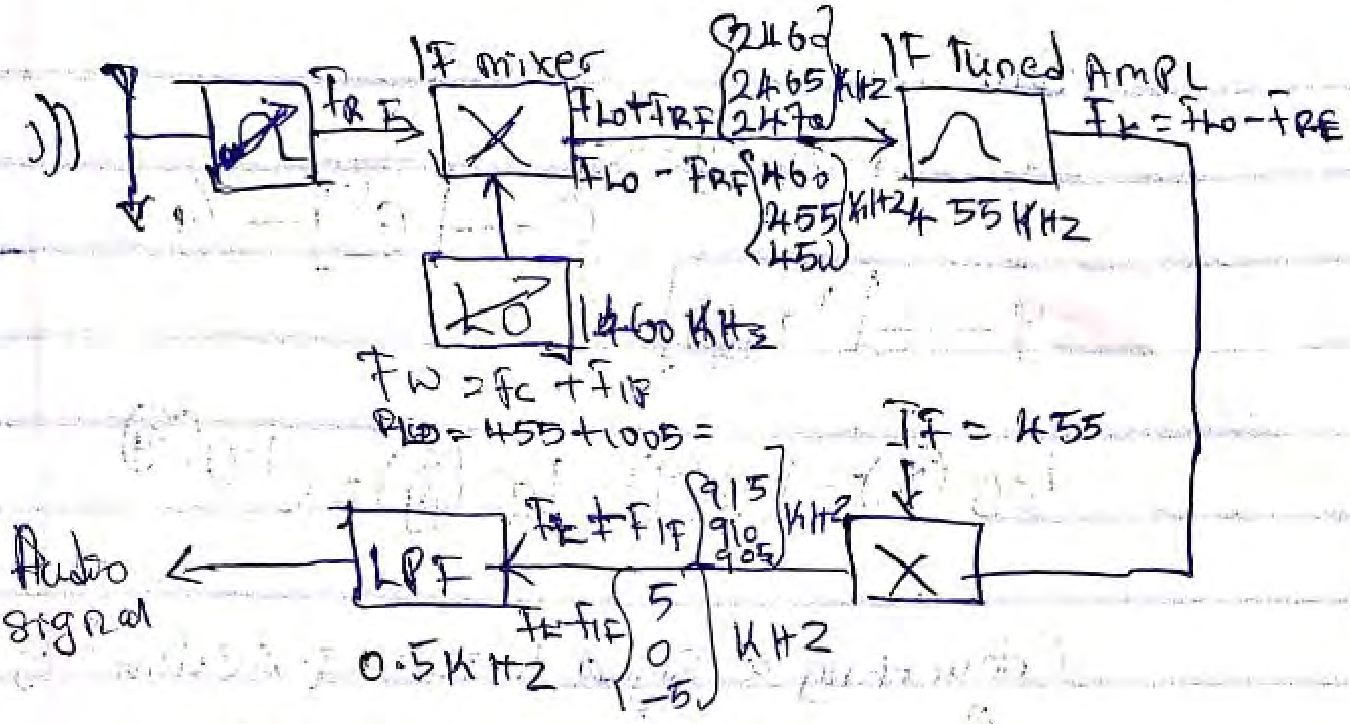
For the difference



Simple operation of an IF Receiver:

AM Broadcasting Station





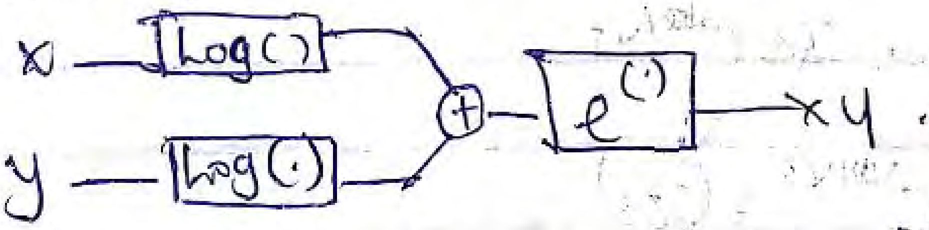
ANALOG MULTIPLIER IMPLEMENTATION

(i) logarithm Amplifier (ii) exponential amplifier.

The logarithm amplifier follows the properties of logarithm which are:

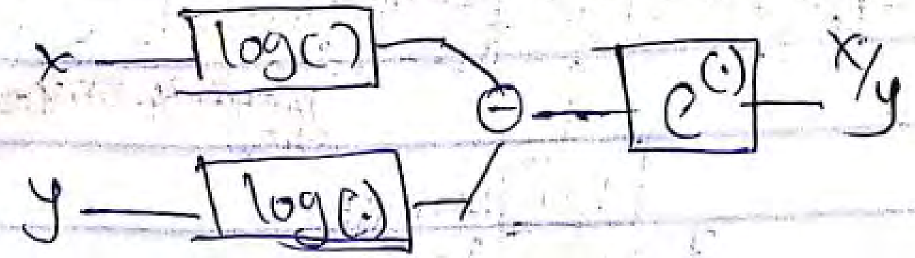
1. $\log(a \cdot b) = \log(a) + \log(b)$
2. $\log(a/b) = \log(a) - \log(b)$

multiplication.



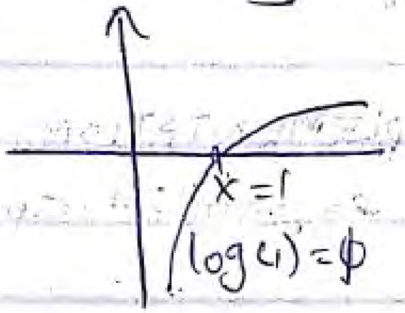
$$\log(x) + \log(y) = \log(xy) = e^{\log(xy)} = xy$$

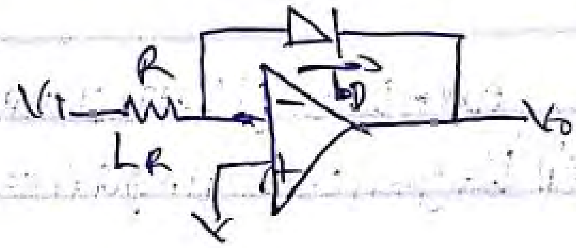
Division:



$$\log(x) - \log(y) = \log(x/y) = e^{\log(x/y)}$$

Converting a signal to its equivalent log value





$$KCL = I_R = I_D \quad \text{— ①}$$

$$\text{Ohm law} = I_R = \frac{V_i}{R} \quad \text{— ②}$$

$$I_D = I_S(e^{V_D/V_T} - 1)$$

$$I_D = I_S e^{V_D/V_T}$$

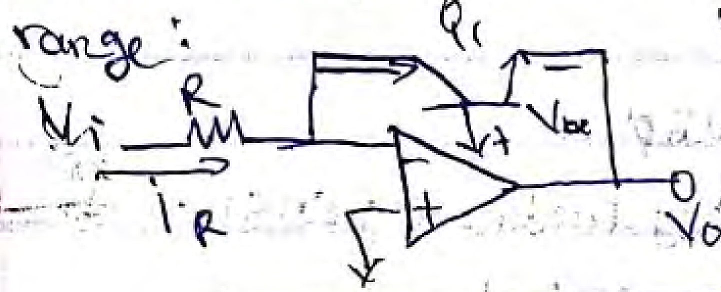
$$\frac{V_i}{R} = I_S e^{V_D/V_T}$$

$$e^{V_D/V_T} = \frac{V_i}{(I_S R)}$$

$$V_D/V_T = \ln(V_i/I_S R)$$

$$V_o = V_T \ln\left[\frac{V_i}{I_S R}\right] \quad V_o = K_1 \cdot \ln\left(\frac{V_i}{I_{S0}}\right)$$

Alternate implementation for improved dynamic range:



$v_o = -V_{be}$

$i_c = I_{s0}\left(e^{V_{be}/V_T} - 1\right)$

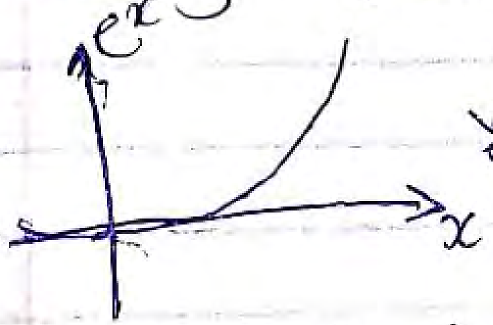
$= I_{s0}\, e^{V_{be}/V_T}$

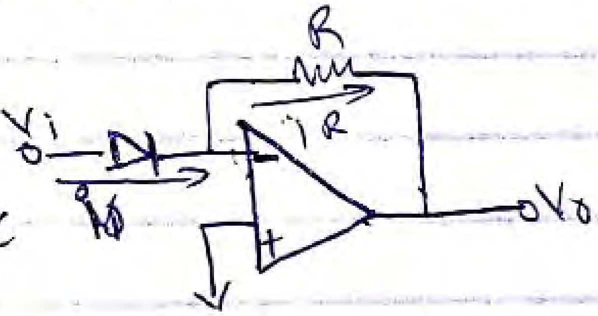
$$\boxed{V_o = -V_T \ln\left(\frac{V_i}{I_{s0}R}\right)}$$

Limitation of this method:-

① Band width ② Single quadrant operation

Antilog Amplifier :- This amplifier calculates the antilog (exponential) function of an input signal





KCL = $i_D = i_R$

Ohm's law $V_o = -i_R \cdot R = i_R = -V_o/R$

Shockeys eq = $i_D = I_S\left(e^{V_D/V_T} - 1\right)$

$i_D = I_S\, e^{V_D/V_T}$

$-\frac{V_o}{R} = I_S e^{V_i/V_T}$

$$\boxed{V_o = -I_S R\, e^{V_i/V_T}}$$

$V_o = K_1\, e^{(V_i/K_2)}$

PHASE LOCKED LOOP Circuits

A PLL System includes, VCO, phase detector, low pass filter. This system allows one oscillator to track with another.

$$\phi_{out}(t) = \phi_{in}(t) + const$$

$$\phi_{out}(t) = \omega_{in}(t)$$

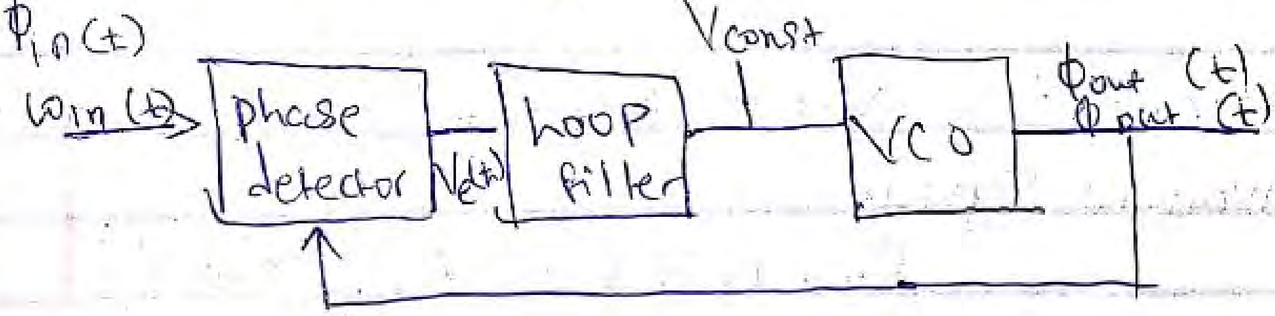
PLL Applications are listed below.

(a) clock generation

(b) frequency synthesizer

(c) clock recovery in a serial data link.

PLL Block Diagram.



Phase detector

The phase detector is a Comparator circuit which Compares the input frequency and VCO output frequency and produces a dc Voltage proportional to the phase difference between the two frequencies.

The phase detector may be analog or digital

$V_e(t)$ = difference (Phase) between two inputs

$$V_e(t) = K_D\,[\phi_{out}(t) - \phi_{in}(t)]$$

~~2.) V.C.O~~ (2.) Low pass filter [LPF]

This is used to get rid of the high frequency components in the output of the phase detector. It also removes the high frequency noise. The LPF helps to control the dynamic characteristics of the whole circuit which includes capture, lock range, bandwidth, & transcient response.

(3.) V.C.O ÷ The V.C.O is treated as a linear time-invariant system

$$\phi_{out} \text{ of V.C.O} = K_O \int_{-\infty}^{t} V_{const}\, dt'$$

which oscillates at an angular frequency of ω_o which is set to nominal when control voltage is zero. frequency is assumed to be linearly proportional to control voltage.

$$\phi_{out} = \omega_o + K_O V_{const}$$

Note ÷ a high loop gain is beneficial for reducing error functio

minimization of phase error at local oscillator

PLL is a feedback system which consist of loop gain of $T(s) = K_{FWD}(s)\, K_{FB}(s)$

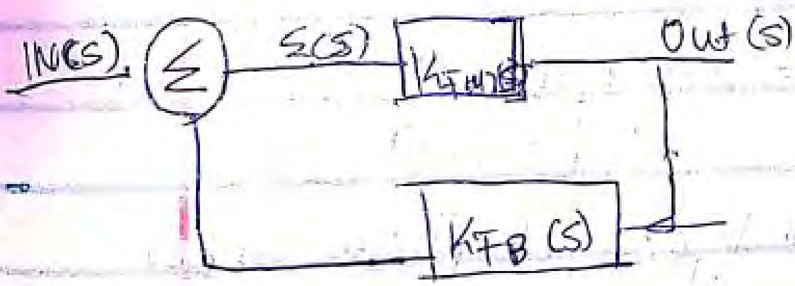
T.F of $\frac{Out(s)}{In(s)} = H(s) = \frac{K_{FWD}(s)}{1+T(s)}$

$T(s) = \frac{K'(s+a)(s+b)}{s^n(s+\alpha)(s+\beta)}$

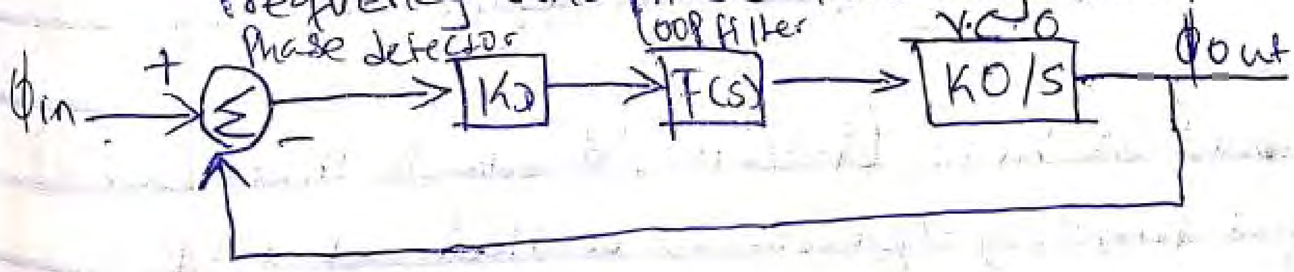
order is in denomination

Type : n (exponent of the s factor in denominator)

Phase error : $\varepsilon(s) = \frac{In(s)}{1+T(s)}$



Frequency and phase tracking loop:



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$H(s) = \text{forward gain} / [1 + T(s)]$.

feedback = 1

$H(s) = T(s) / [1 + T(s)]$

$$H(s) = \frac{\phi_{out}}{\phi_{in}} = \frac{K_D K_0 f(s)/s}{1 + K_D K_0 f(s)/s}$$

Phase error function

$$\varepsilon_s = \phi_{in} - \phi_{out} = \frac{S\phi_{in}}{S + K_D K_0 F(s)}$$

PHASE ERROR.

There is no frequency error when the loop is locked because the input frequency = output frequency.

$$\text{phase Error} = \varepsilon(s) = \frac{IN(s)}{1 + T(s)}$$

we have two types of phase Error

① Steady State error

$$\varepsilon_{ss} = \lim_{s \to 0} [S\varepsilon(s)] = \lim_{t \to \infty} \varepsilon(t)$$

from Gardner's book $\varepsilon(s) = (20 \log(\phi_{out}/\phi_{in})$ dB which increases as the input frequency approaches a natural loop frequency $\zeta = 0.707$

Transcient phase Error :- This is the inverse laplace transform of $\varepsilon(s)$ (i.e Steady State error). It has three steps ~~3 forms~~ which are stated below:

(1) Phase Step :- Because $\phi_{in}(t) = \Delta\theta\, u(t)$ in the frequency domain hence

$$\phi_{in}(s) = \frac{\Delta\theta}{s}$$

Hence $$\varepsilon_{ss} = \lim_{s \to 0} \frac{s\,\frac{\Delta\theta}{s}}{1 + \frac{K_v}{s}\left[\frac{1 + s/\omega_2}{1 + s/\omega_1}\right]} = 0$$

Control voltage must return to same value after phase step is complete Hence there is only a transient phase error for a phase step.

(2) frequency ramp :- This type of Error gives unlimited steady state error hence it is not suitable for tracking a moving source.

$$F(s) = \frac{1 + s/\omega_2}{1 + s/\omega_1} \quad \text{or} \quad F(s) = \frac{1 + s/\omega_2}{s/\omega_1}$$

B) Frequency Step :-

$$\varepsilon_{ss} = \lim_{s \to 0} \frac{s\frac{\Delta\omega}{s^2}}{1 + \frac{K_v}{s}\left(\frac{1+s/\omega_2}{1+s/\omega_1}\right)} = \lim_{s \to 0} \frac{\Delta\omega}{s + K_v\left(\frac{1+s/\omega_2}{1+s/\omega_1}\right)} = \frac{\Delta\omega}{K_v}$$

This includes static error but ~~it~~ can be made small by increasing K_v value. which is consistent with the idea that a shift in control voltage is need to give a frequency step up.

PLL Phase Noise

we have two main source of noise in the phase lock loop:

① reference noise - ② VCO noise

Reference noise

$$\frac{\phi_{out}}{\phi_{ref}} = \frac{\text{forward path}}{1+T(s)} = \frac{K_v F(s)/s}{1+K_v F(s)/Ns}$$

$$= \frac{N(1+s/\omega_2)}{Ns^2/K_v + s/\omega_2 + 1}$$

~~Ad: limitat~~ion