NAME : OKOLI TESSY EBUBE
DEPARTMENT : MBBS
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COVID-19 HOLIDAY ASSIGNMENT

SECTION A

1. Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

Answer

CHARGING BY INDUCTION

Electric charges can be obtained on an object without touching it, by a process called Electrostatic Induction.

Consider a positively charged rubber rod brought near a neutral [unchanged] conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod [Fig. 1]. The region of the sphere nearest the positively charged rod has an excess of negative charge because of the migration of protons away from this location. If a grounded conducting wire is then connected to the sphere, as in [Fig. 2], some of the protons leave the sphere and travel to the earth. If the wire to ground is then removed [Fig. 3], the conducting sphere is left with an excess of induced negative charge.

Finally, when the rubber rod is removed from the vicinity of the sphere [Fig. 4], the induced negative charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.

Diagrams:

Fig 1

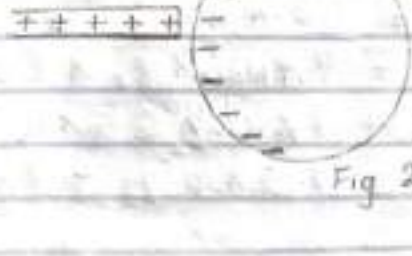

Fig 2

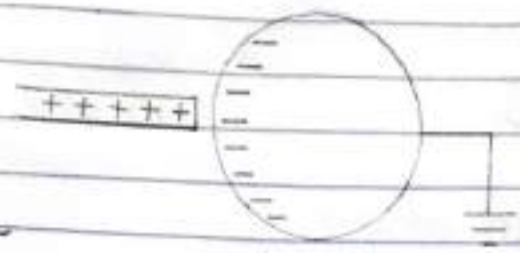

Fig. 3

Fig. 4

b Each of two small spheres is charged positively, the combined cha[rge] being $5.0 \times 10^{-5}C$. If each sphere is repelled from the other by a force of $1.0N$ when the spheres are $2.0m$ apart, Calculate the charge on each sphere.

Solution

$K = 9 \times 10^{9} Nm^2C^{-2}$

$q_1 = ?$

$q_2 = ?$

$F = 1.0N$

$r = 2.0m$

$q_1 + q_2 = 5.0 \times 10^{-5}C$

$\therefore q_1 = 5.0 \times 10^{-5}C - q_2$

Using Coulomb's Law

$$F = \frac{Kq_1q_2}{r^2}$$

$$\frac{1.0}{1} = \frac{9\times10^{9} \times (5\times10^{-5} - q_2) \times q_2}{(2.0)^2}$$

$$\frac{1.0}{1} = \frac{9\times10^{9} \times q_2 \times (5\times10^{-5} - q_2)}{4}$$

$$4 = 9\times10^{9} \times q_2 \times (5\times10^{-5} - q_2)$$

$$4 = 9\times10^{9} q_2 (5\times10^{-5} - q_2)$$

$$4 = 4.5\times10^{5} q_2 - 9\times10^{9} q_2^2$$

$$9\times10^{9} q_2^2 - 4.5\times10^{5} q_2 + 4 = 0$$

Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 9 \times 10^{-9}$, $b = -4.5 \times 10^5$, $c = +4$

$$q_2 = \frac{-(-4.5 \times 10^5) \pm \sqrt{(-4.5 \times 10^5)^2 - 4 \times 9 \times 10^9 \times 4}}{2 \times (9 \times 10^9)}$$

$$q_2 = \frac{4.5 \times 10^5 \pm \sqrt{2.025 \times 10^{11} - 1.44 \times 10^{11}}}{1.8 \times 10^{10}}$$

$$q_2 = \frac{4.5 \times 10^5 \pm \sqrt{5.85 \times 10^{10}}}{1.8 \times 10^{10}}$$

$$q_2 = \frac{4.5 \times 10^5 + \sqrt{5.85 \times 10^{10}}}{1.8 \times 10^{10}}$$

$$q_2 = \frac{4.5 \times 10^5 + 241867.7234}{1.8 \times 10^{10}}$$

$$q_2 = \frac{691867.7324}{1.8 \times 10^{10}}$$

$q_2 = 3.8437 \times 10^{-5} \Omega$

$q_2 = 3.84 \times 10^{-5} C$

$$q_2 = \frac{4.5 \times 10^5 - \sqrt{5.85 \times 10^{10}}}{1.8 \times 10^{10}}$$

$$q_2 = \frac{4.5 \times 10^5 - 241867.7234}{1.8 \times 10^{10}}$$

$$q_2 = \frac{208132.2766}{1.8 \times 10^{10}}$$

$q_2 = 1.15629 \times 10^{-5} \Omega$

$q_2 = 1.16 \times 10^{-5} C$

Recall,

$q_1 = 5 \times 10^{-5} - q_2$

when $q_2 = 3.84 \times 10^{-5} C$

$q_1 = 5 \times 10^{-5} - 3.84 \times 10^{-5}$

$q_1 = 1.16 \times 10^{-5} C$

when $q_2 = 1.16 \times 10^{-5} C$

$q_1 = 5 \times 10^{-5} - 1.16 \times 10^{-5}$

$q_1 = 3.84 \times 10^{-5} C$

$\therefore$ $q_1 = 1.16 \times 10^{-5} C$ or $3.84 \times 10^{-5} C$ and $q_2 = 3.84 \times 10^{-5} C$ or $1.16 \times 10^{-5} C$

c. Three charges were positioned as shown in the figure below. If $Q_1 = Q_2 = 8 \mu C$ and $d = 0.5 m$, determine q if the electric field at P is zero

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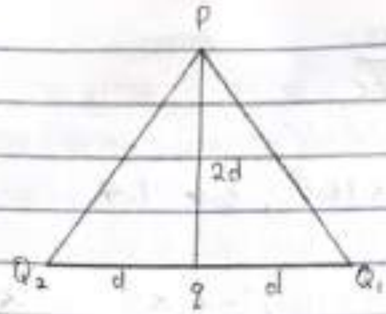
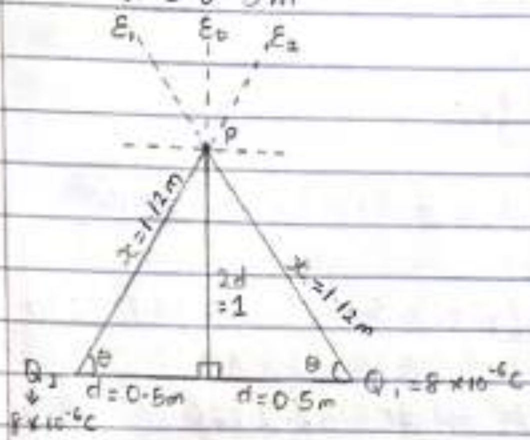


Solution.

$Q_1 = Q_2 = 8\mu C = 8\times10^{-6}C$

$d = 0.5m$



To find x, using Pythagora's theo

$x^2 = 2d^2 + d^2$

$x^2 = 1^2 + 0.5^2$

$x = \sqrt{1^2 + 0.5^2}$

$x = \sqrt{1 + 0.25}$

$x = \sqrt{1.25}$

$\therefore x = 1.12m$

To find θ, using SOHCAHTOA

$Tan\theta = \frac{1}{0.5}$

$Tan\theta = 2$

$\theta = Tan^{-1}(2)$

$\theta = 63.43°$

$E = \frac{Kq}{r^2}$

$Q_1 = Q_2$, NOTE: $E_1 = E_2$

$E_1 = E_2 = \frac{9\times10^9 Nm^2C^{-2} \times 8\times10^{-6}C}{(1.12m)^2}$

$E_1 = E_2 = \frac{7200}{1.2544} = 5739.795918$

$E_1 = E_2 = 5.74\times10^4 NC^{-1}$

$E_q = \frac{kq}{r^2} = \frac{9\times10^9 Nm^2C^{-2} \times q}{1^2} = 9\times10^9 q\,NC^{-1}$

$\therefore E_q = 9\times10^9 q\,NC^{-1}$

FORCES	ANGLES	x - component [$E\cos\theta$]	y - component [$E\sin\theta$]
$5.74\times10^4 NC^{-1}$	$63.43°$	$5.74\times10^4 \cos 63.43°$	$5.74\times10^4 \sin 63.43$
		$= +2.57\times10^4$	$= 5.13\times10^4$
$5.74\times10^4 NC^{-1}$	$63.43°$	$5.74\times10^4 \cos 63.43°$	$5.74\times10^4 \sin 63.43$
		$= -2.57\times10^4$	$= 5.13\times10^4$
$9\times10^9_q NC^{-1}$	$90°$	$9\times10^9_q \cos 90° = 0$	$9\times10^9_q \sin 90° = 9.0\times10^9 q$
		$\Sigma x = 0 NC^{-1}$	$\Sigma y = 1.026\times10^5 NC^{-1} + 9.0\times10^9 q$

Magnitude $= \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

$E_q = \sqrt{\Sigma_x^2 + \Sigma_y^2}$

$E_q = \sqrt{(0^2) + [(1.026\times10^5)^2 + (9.0\times10^9 q)^2]}$

$E_q = \sqrt{1.053\times10^{10} + (9.0\times10^9 q)^2}$

But $E_q = 0$

$\therefore 0 = \sqrt{1.053\times10^{10} + (9.0\times10^9 q)^2}$

Square both sides

$0^2 = \left(\sqrt{1.053\times10^{10} + (9.0\times10^9 q)^2}\right)^2$

$0 = 1.053\times10^{10} + (9.0\times10^9 q)^2$

$(-9.0\times10^9 q)^2 = 1.053\times10^{10}$

$\therefore q^2 = \frac{1.053\times10^{10}}{(-9.0\times10^9)^2} = \frac{1.053\times10^{10}}{8.1\times10^{19}} = 1.3\times10^{-10} C$

$q = \sqrt{1.3\times10^{-10}}$

$q = 1.14\times10^{-5} C$.

$\therefore q = 11.4 \mu C$

2. Distinguish between the terms of electric field and electric field intensity.

Electric Field is the region of space in which an electric charge will

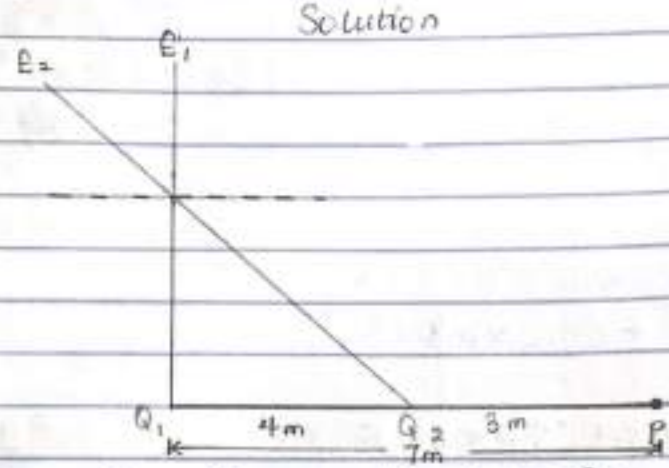
will experience an electric force while **electric** field intensity is also known as "electric field strength" is defined as the force per unit **charge**; it is the magnitude of electric field.

2b. A positive charge $Q_1 = 8nC$ is at the origin, and a second positive charge $Q_2 = 12nC$ is on the x-axis at $x = 4m$. Find

i) the net electric field at a point P on the x-axis at $x = 7m$

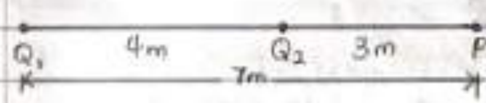
ii) the electric field at a point Q on the y axis at $y = 3m$ due to the charges

Solution



$Q_1 = 8nC$, $Q_2 = 12nC \Rightarrow Q_1 = 8 \times 10^{-9}C$, $Q_2 = 12 \times 10^{-9}C$

i)



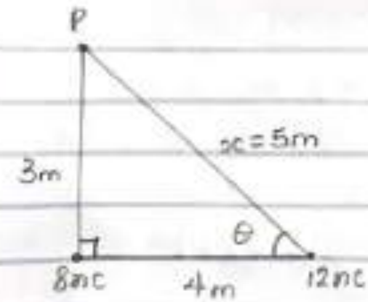
$$E_1 = \frac{KQ_1}{r_1^2} = \frac{9 \times 10^9 Nm^2C^{-2} \times 8 \times 10^{-9}C}{7^2} = 1.469NC^{-1}$$

$$E_2 = \frac{KQ_2}{r^2} = \frac{9 \times 10^9 Nm^2C^{-2} \times 12 \times 10^{-9}C}{3^2} = 12NC^{-1}$$

$$E_{net} = E_1 + E_2 = 1.469NC^{-1} + 12NC^{-1} = 13.469NC^{-1} \simeq 13.5NC^{-1}$$

$\therefore E_{net} = 13.5NC^{-1}$

bii



To find x using Pythagora's theorem

$x^2 = 3^2 + 4^2$

$x = \sqrt{3^2 + 4^2}$

$x = \sqrt{9 + 16} = \sqrt{25}$

$\therefore x = 5m$

To find θ using SOHCAHTOA

$\tan\theta = \frac{3}{4}$

$\tan\theta = 0.75$

$\theta = \tan^{-1}(0.75) = 36.9°$

$E_1 = \frac{kQ_1}{r_1^2} = \frac{9 \times 10^9 Nm^2C^{-2} \times 8 \times 10^{-9}C}{3^2} = 8NC^{-1}$

$E_2 = \frac{kQ_2}{r_2^2} = \frac{9 \times 10^9 Nm^2C^{-2} \times 12 \times 10^{-9}C}{5^2} = 4.32NC^{-1}$

Vector	Angle	x-component	y-component
$8NC^{-1}$	$90°$	$8NC^{-1}\cos 90 = 0NC^{-1}$	$8\sin 90° = 8NC^{-1}$
$4.32NC^{-1}$	$36.9°$	$4.32\cos 36.9 = 3.45NC^{-1}$	$4.32\sin 36.9 = 2.60NC^{-1}$
		$\Sigma fx = 3.45NC^{-1}$	$\Sigma fy = 10.60NC^{-1}$

$\Sigma net = \sqrt{\Sigma fx^2 + \Sigma fy^2}$

$\Sigma net = \sqrt{3.45^2 + 10.60^2}$

$\Sigma net = \sqrt{124.2625}$

$\Sigma net = 11.147NC^{-1} \simeq 11.5NC^{-1}$

SECTION B

4a What is Magnetic flux?

Magnetic Flux refers to the number of magnetic lines of force passing through a given closed surface which is the magnetic field. It is what generates the field around a magnetic material. Its S.I unit is Weber (Wb). It is also the strength of a magnetic field ~~when~~ represented by lines of force and usually denoted by the symbol ϕ.

b An electron with a rest mass of 9.11×10^{-31} kg moves in a circ orbit of radius 1.4×10^{-7} m in a uniform magnetic field of 3.5×10 Weber/meter square, perpendicular to the speed with which elec moves. Find the cyclotron frequency of the moving electron.

Solution

$m_e = 9.11 \times 10^{-31} kg$ $\quad \omega = ?$

$r = 1.4 \times 10^{-7} m$

$\theta = 90^{\circ}$

$B = 3.5 \times 10^{-1} Wb/m^2$

$q = 1.6 \times 10^{-19} C$

Cyclotron frequency, $\omega = \frac{qB}{m_e} = \frac{v}{r}$

$$\omega = \frac{1.6 \times 10^{-19} C \times 3.5 \times 10^{-1} Wb/m^2}{9.11 \times 10^{-31} kg}$$

$$\omega = \frac{5.6 \times 10^{-20}}{9.11 \times 10^{-31}}$$

$$\omega = 6.147 \times 10^{10} \approx 6.15 \times 10^{10}$$

$$\omega = 6.15 \times 10^{10} rad/s$$

c. Discuss your answer in 4b above

The cyclotron frequency is the inverse of the period which is the time taken for the accelerated electron to complete a cycle in the magnetic field.

The cyclotron frequency is also known as the angular speed of the particle

5a State the Biot-Savart Law.

Biot-Savart Law states that the magnetic field is directly proportional to the product permeability of free space (μ), the current (I), the change in length, the radius and inversely proportional to the square of radius (r^2). It can be expressed mathematically by,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \hat{r}}{r^2}$$

where μ_0 is a constant called Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} T.m/A$$

The unit of $\vec{B}$ is Weber/metre square

b. Using the Biot - Savart Law, show that the magnitude of the magnetic field of a straight current - carrying conductor is given as

$$B = \frac{\mu_0 I}{2\pi r}$$

Answer.

MAGNETIC FIELD OF A STRAIGHT CURRENT-CARRYING CONDUCTOR

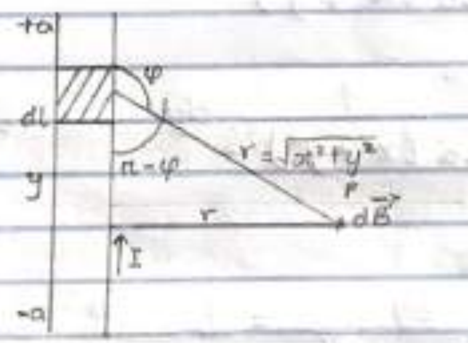


Diagram of a section of a straight current carrying conductor.

Applying the Biot-Savart Law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad ---(*)$$

$$\text{But } \sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}} \quad ---(**)$$

Substituting $(**)$ into $(*)$, we have

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad ---(***)$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

Equation $(***)$ therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is, when a is much larger than x,

$$(x^2 + a^2)^{1/2} \cong a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y-axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{---}(\#)$$

Equation (#) defines the magnitude of the magnetic field of flux density B near a long, straight current carrying conductor.