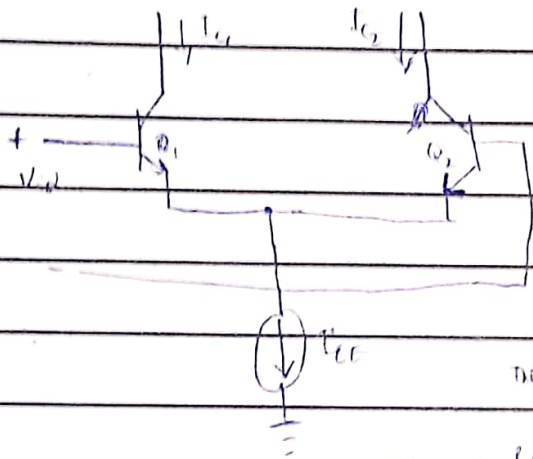


Analog multipliers

These are probably the most important analog signal processing blocks. It is a circuit that produces an output which is proportional to the product of its inputs.

The emitter-coupled pair as a simple multiplier:



As shown it produces output currents related to the differential input voltage by:

$$I_{C1} = \frac{I_{EE}}{1 + \exp(-V_{id}/V_T)}$$

$$I_{C2} = \frac{I_{EE}}{1 + \exp(V_{id}/V_T)}$$

$$I_{EE} = I_{EE} \tanh(V_{id}/2V_T)$$

The differential output current $\Delta I_C = I_{C1} - I_{C2} = I_{EE} \tanh(V_{id}/2V_T)$

assuming $(V_{id}/2V_T) \ll 1$: $\Delta I_C \approx I_{EE} (V_{id}/2V_T)$

As with the addition of more circuitry, I_{EE} which is the bias current for the emitter-coupled pair becomes proportional to a second input signal: therefore, $I_{EE} \approx K_0 (V_{i2} - V_{BE(on)})$

The differential output current of the emitter-coupled pair also becomes

$$\Delta I_C \approx \frac{K_0 V_{id} (V_{i2} - V_{BE(on)})}{2V_T}$$

The produced circuit functions as a multiplier under the assumption that V_{id} is small and $V_{i2} > V_{BE(on)}$.

later

The latter assumption means the multiplier only functions in 2 quadrants of the $V_{id} - V_{i2}$ plane and this type of circuit is called a two-quadrant multiplier.

The Gilbert multiplier cell: This is a modification of the emitter-coupled cell and it allows four-quadrant multiplication. It is the series connection of ~~an emitter-coupled pair with two~~ two cross-coupled emitter-coupled pair with an emitter-coupled pair.

The collector current of Q_3 & Q_4 are given by

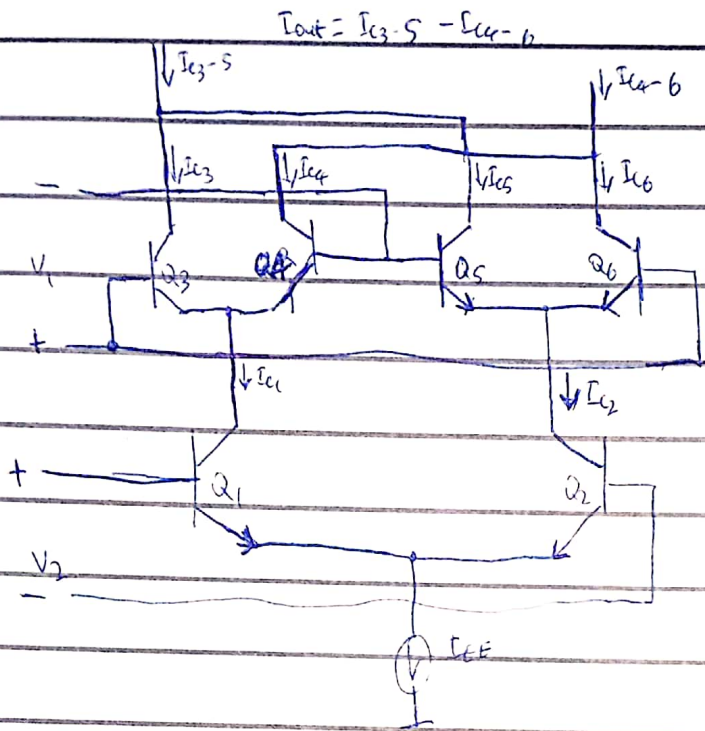
as Q_5 & Q_6

$$I_{C3} = \frac{I_{C1}}{1 + \exp(-V_1/V_T)}$$

$$I_{C4} = \frac{I_{C1}}{1 + \exp(V_1/V_T)}$$

$$I_{C5} = \frac{I_{C2}}{1 + \exp(V_2/V_T)}$$

$$I_{C6} = \frac{I_{C2}}{1 + \exp(-V_2/V_T)}$$



The currents I_{C1} & I_{C2} are related to V_2

$$I_{C1} = \frac{I_{EE}}{1 + e^{-V_2/V_T}} \quad \& \quad I_{C2} = \frac{I_{EE}}{1 + e^{V_2/V_T}}$$

After substituting I_{C1} & I_{C2} into I_{C3} , I_{C4} , I_{C5} & I_{C6} - The differential output current is ^{then} given by

$$\Delta I = I_{C3-5} - I_{C4-6} = I_{C3} + I_{C5} - (I_{C4} + I_{C6}) = I_{EE} \tanh(V_1/2V_T) \tanh(V_2/2V_T)$$

The Gilbert cell only acts as a multiplier when $V_1 \ll V_T$ & $V_2 \ll V_T$. That is, it performs analog multiplication

for small-amplitude signals.

Amplitudes of input signals are often much larger than V_T . So, a nonlinearity can be introduced to pre-distort

the input signals to compensate for the hyperbolic tangent transfer characteristic of the basic cell. The required

nonlinearity is called an inverse hyperbolic tangent

$$I_1 = I_{O1} + K_1 V \quad \& \quad I_2 = I_{O1} - K_1 V$$

where I_{O1} is the dc that flows in each output lead if $V_1 = 0$

K = transconductance of the voltage-to-current converters.

$$\Delta I = I_{EE} \begin{pmatrix} K_1 V_1 \\ I_{O1} \end{pmatrix} \begin{pmatrix} K_2 V_2 \\ I_{O2} \end{pmatrix}$$

$$V_{out} = I_{EE} K_1 K_2 \frac{V_1 V_2}{I_{O1} I_{O2}} = 0.1 V_1 V_2$$

Gilbert cell as a balanced modulator

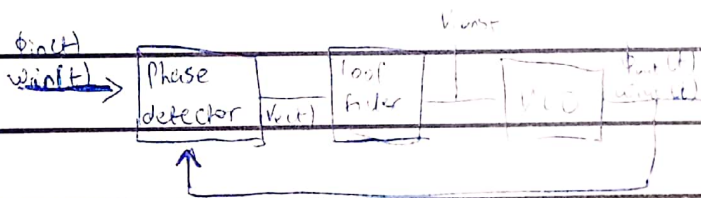
Looking at the Gilbert cell; if one of the inputs of a signal is large compared to V_T , it effectively multiplies the applied small voltage signal by a square wave making the cell a modulator.

Gilbert cell as a Phase detector

If the two inputs are large compared to V_T and are both unmodulated signals of similar frequency, the circuit behaves as a phase detector and produces an output whose dc component is proportional to the phase difference between the 2 inputs.

Phase locked loop circuits

A phase locked loop is a feedback system that includes a phase detector, VCO & low pass filter within its loop. It is used to force the VCO to replicate & track the frequency & phase at the input when in lock. It allows one oscillator to track with another.



Phase & frequency are interrelated by:

$$\omega(t) = \frac{d\phi}{dt}$$

$$\phi(t) = \phi(0) + \int_0^t \omega(t') dt'$$

Phase detector: This compares the phase at each input & generates an error signal, $V_e(t)$, proportional to the phase difference between the two inputs. K_D is the gain of the phase detector (V/rad).

$$V_e(t) = K_D (\phi_{out}(t) - \phi_{in}(t))$$

Since 2 inputs are at the same frequency when the loop is locked, we have one output at twice the input frequency & an output proportional to the cosine of the phase difference. The doubled frequency ^{component} is then removed by the low pass filter. Any phase difference can then show up as the control voltage to the VCO.

VCO: Excess phase of the VCO is the system output

$$\phi_{out} = K_O \int_{-\infty}^t V_{cont} dt'$$