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18/SCI01/047

MAT 204

$$A = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{pmatrix}$$

(i) Rank of A

$$|A| = \begin{vmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} - 3 \begin{vmatrix} 4 & 2 \\ 8 & 1 \end{vmatrix} + 6 \begin{vmatrix} 4 & 0 \\ 8 & 5 \end{vmatrix}$$

$$= 1(0 - 10) + 3(4 - 16) - 6(20 - 0)$$

$$= -10 - 36 - 120$$

$$|A| = -166$$

\therefore The rank of A is 3

(ii) Rank of B^T

$$B^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{pmatrix}$$

$$|B^T| = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -2 & 3 \\ 1 & -4 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + 0$$

$$= 1(8 - 3) - 1(-4 - 6)$$

$$= 2 - 5 + 10$$

$$|B^T| = 15$$

∴ The Rank of B^T is 3

(iii) Rank of $(A+C)^T$

$$(A+C) = \begin{pmatrix} 1+0 & -3+4 & 6+3 \\ 4+6 & 0-7 & 2+1 \\ 8+1 & 5-3 & 1+2 \end{pmatrix}$$

$$(A+C) = \begin{pmatrix} 1 & 1 & 9 \\ 10 & -7 & 3 \\ 9 & 2 & 3 \end{pmatrix} \quad (A+C)^T = \begin{pmatrix} 1 & 10 & 9 \\ 1 & -7 & 2 \\ 9 & 3 & 3 \end{pmatrix}$$

$$|A+C^T| = \begin{vmatrix} 1 & 10 & 9 \\ 1 & -7 & 2 \\ 9 & 3 & 3 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -7 & 2 \\ 3 & 3 \end{vmatrix} - 10 \begin{vmatrix} 1 & 2 \\ 9 & 3 \end{vmatrix} + 9 \begin{vmatrix} 1 & -7 \\ 9 & 3 \end{vmatrix}$$

$$= 1(-21 - 6) - 10(3 - 18) + 9(3 + 63)$$

$$= -27 + 150 + 594$$

$$|A+C^T| = 717$$

∴ The Rank of $(A+C^T)$ is 3

iv) Rank of $(B+C)$

$$(B+C) = \begin{pmatrix} 1+0 & 1+4 & 2+3 \\ 1+6 & -2-7 & 1+1 \\ 0+1 & 3-3 & -4+2 \end{pmatrix}$$

$$(B+C) = \begin{pmatrix} 1 & 5 & 5 \\ 7 & -9 & 2 \\ 1 & 0 & -2 \end{pmatrix}$$

$$|B+C| = \begin{vmatrix} 1 & 5 & 5 \\ 7 & -9 & 2 \\ 1 & 0 & -2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} -9 & 2 \\ 0 & -2 \end{vmatrix} - 5 \begin{vmatrix} 7 & 2 \\ 1 & -2 \end{vmatrix} + 5 \begin{vmatrix} 7 & -9 \\ 1 & 0 \end{vmatrix}$$

$$= 1(18-0) - 5(-14-2) + 5(0+9)$$
$$= 18 + 80 + 45$$

$$|B+C| = 143$$

∴ The Rank of $(B+C)$ is 3

v) Rank of $(A+B+C)$

$$(A+B+C) = \begin{pmatrix} 1+1+0 & -3+1+4 & 6+2+3 \\ 4+1+6 & 0-2-7 & 2+1+1 \\ 8+0+1 & 5+3-3 & 1-4+2 \end{pmatrix}$$

$$(A+B+C) = \begin{pmatrix} 2 & 2 & 11 \\ 11 & -9 & 5 \\ 9 & 5 & -1 \end{pmatrix}$$

$$|A+B+C| = \begin{vmatrix} 2 & 2 & -9 \\ 5 & -2 & 11 \\ 5 & -1 & 5 \end{vmatrix} + 11 \begin{vmatrix} 2 & 2 \\ 5 & 5 \end{vmatrix} + 11 \begin{vmatrix} 2 & 2 \\ 5 & 5 \end{vmatrix}$$

$$= 2(9 - 25) - 2(-11 - 45) + 11(55 - 10)$$

$$= -32 - 2(-56) + 11(45)$$

$$= -32 + 112 + 495 = 575$$

∴ The Rank of $(A+B+C)$ is 3

$$\begin{vmatrix} 2 & 2 & -9 \\ 5 & -2 & 11 \\ 5 & -1 & 5 \end{vmatrix} = 0$$

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$$2(9 - 25) - 2(-11 - 45) + 11(55 - 10) = 0$$

$$-32 + 112 + 495 = 0$$

$$575 = 0$$

∴ Rank of $(A+B+C)$ is 3

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$$(A+B+C) = \begin{pmatrix} 2 & 2 & -9 \\ 5 & -2 & 11 \\ 5 & -1 & 5 \end{pmatrix}$$

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