

Section A

6. Electric charges can be obtained on an object without touching it, by a process called electrostatic induction. Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere furthest away from the rod (Fig 1.3a). The region of the sphere nearest the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere, as in (Fig 1.3b), some of the electrons leave the sphere and travel to the earth. If the wire is then removed (Fig 1.3c), the conducting sphere is left with an excess of induced positive charge. Finally, when the rubber rod is removed from the vicinity of the sphere (Fig 1.3d), the ~~conducting~~ sphere is left with an excess of induced positive charge and becomes uniformly distributed over the surface of the sphere.

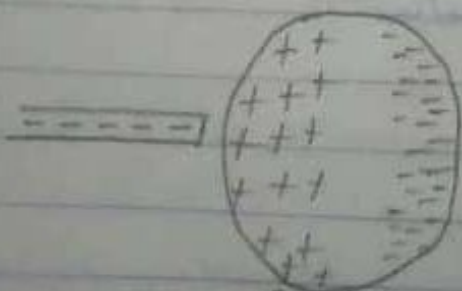


Fig 1.3a

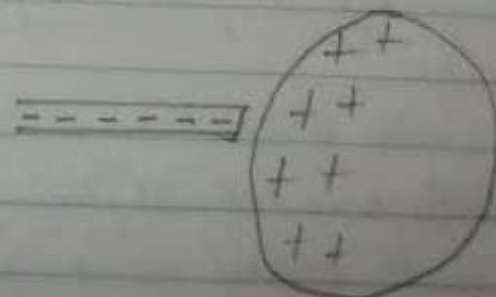


Fig 1.3b

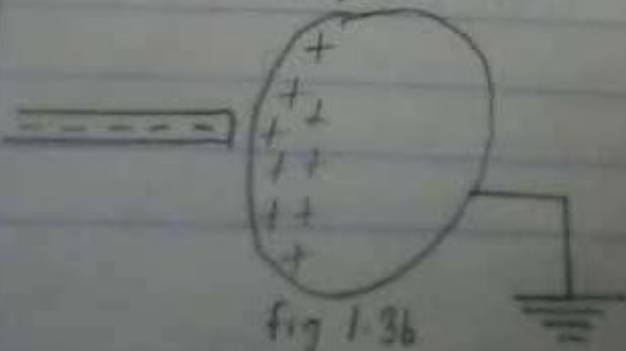


Fig 1.3c

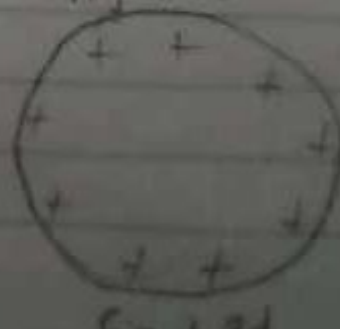


Fig 1.3d

$$q_1 + q_2 = 5.0 \times 10^{-5} \text{ C}$$

$$r = 2 \text{ cm}$$

$$F = 1.0 \text{ N}$$

$$q_1 = ? \text{ and } q_2 = ?$$

$$\text{But } F = \frac{k q_1 q_2}{r^2}$$

$$1 = \frac{9 \times 10^9 \times q_1 q_2}{(2)^2}$$

$$4 = 9 \times 10^9 \times q_1 q_2$$

$$q_1 q_2 = 4.44 \times 10^{-10}$$

$$\therefore q_1 = \frac{4.44 \times 10^{-10}}{q_2}$$

$$q_1 + q_2 = 5 \times 10^{-5}$$

$$\frac{4.44 \times 10^{-10}}{q_2} + q_2 = 5 \times 10^{-5}$$

$$4.44 \times 10^{-10} + q_2^2 = 5 \times 10^{-5} q_2$$

$$q_2^2 - 5 \times 10^{-5} q_2 + 4.44 \times 10^{-10} = 0$$

$$\text{Using formula method } q_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q_2 = \frac{-(-5 \times 10^{-5}) \pm \sqrt{(-5 \times 10^{-5})^2 - 4(1)(4.44 \times 10^{-10})}}{2(1)}$$

$$q_2 = \frac{5 \times 10^{-5} \pm 2.68 \times 10^{-5}}{2}$$

$$q_2 = \frac{5 \times 10^{-5} + 2.68 \times 10^{-5}}{2} \quad \text{or} \quad \frac{5 \times 10^{-5} - 2.68 \times 10^{-5}}{2}$$

$$q_2 = 3.84 \times 10^{-5} \text{ C} \quad \text{or} \quad 1.16 \times 10^{-5} \text{ C}$$

$$\text{For } q_2 = 3.84 \times 10^{-5} \text{ C} \quad \text{For } q_2 = 1.16 \times 10^{-5} \text{ C}$$

$$q_1 = 5.0 \times 10^{-5} - 3.84 \times 10^{-5} \quad q_1 = 5.0 \times 10^{-5} - 1.16 \times 10^{-5}$$

$$q_1 = 1.16 \times 10^{-5} \text{ C} \quad q_1 = 3.84 \times 10^{-5} \text{ C}$$

$$\therefore q_1 = 1.16 \times 10^{-5} \text{ C and } q_2 = 3.84 \times 10^{-5} \text{ C}$$

or vice versa.

$$E_{Q_2} = \frac{kQ_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{4^2} = 12 \text{ NC}^{-1}$$

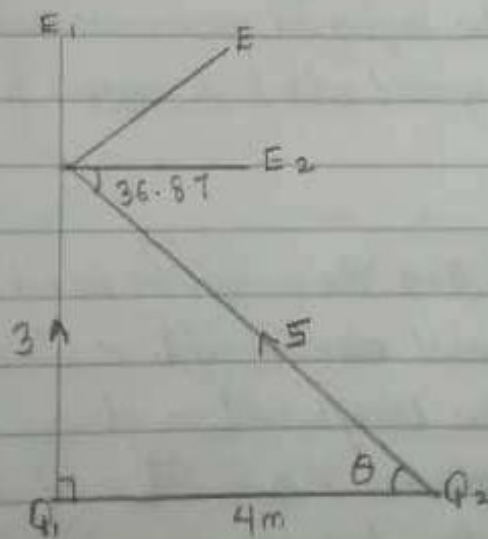
Vector	Angle	X-component	Y-component
$E_{Q_1} = 1.47$	0	$1.47 \cos 0 = 1.47$	$1.47 \sin 0 = 0$
$E_{Q_2} = 12$	0	$12 \cos 0 = 12$	$12 \sin 0 = 0$
		$\Sigma x = 13.47$	$E_y = 0$

$$E_{\text{net}} = \sqrt{(13.47)^2 + (0)^2}$$

$$E_{\text{net}} = 13.47 \text{ NC}^{-1}$$

$$\text{ii) } E_2 = \frac{kQ_2}{r^2} = \frac{(9 \times 10^9)(12 \times 10^{-9})}{5^2} = 4.32 \text{ NC}^{-1}$$

$$E_1 = \frac{kQ_1}{r^2} = \frac{(9 \times 10^9)(3 \times 10^{-9})}{3^2} = 8 \text{ NC}^{-1}$$



$$\tan \theta = \frac{3}{4}$$

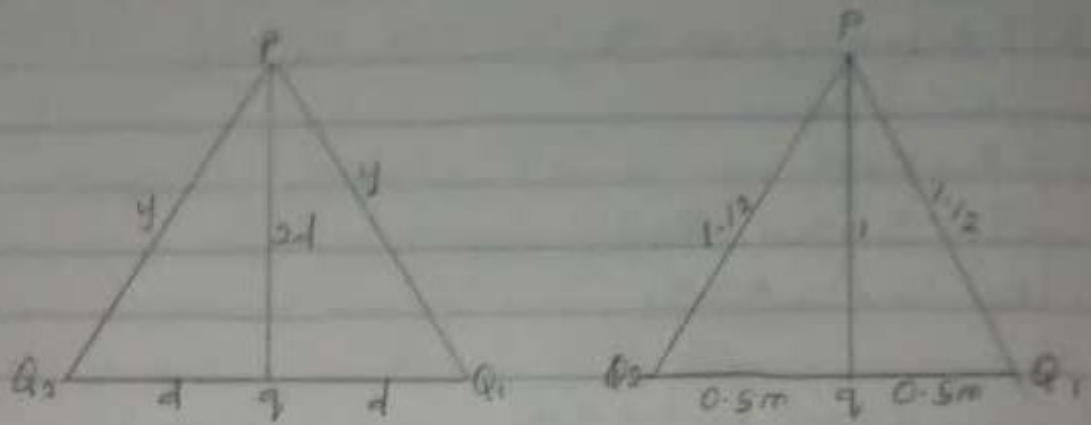
$$\theta = \tan^{-1}(0.75) = 36.87$$

Vector	Angle	X-component	Y-component
$E_2 = 4.32$	36.87	$4.32 \cos 36.87 = 3.46$	$4.32 \sin 36.87 = 2.57$
$E_1 = 8$	90	$8 \cos 90 = 0$	$8 \sin 90 = 8$
		$\Sigma x = 3.46$	$\Sigma y = 10.57$

$$E_{\text{net}} = \sqrt{\Sigma x^2 + \Sigma y^2}$$

$$E_{\text{net}} = \sqrt{(3.46)^2 + (10.57)^2}$$

$$E_{\text{net}} = 11.14 \text{ NC}^{-1}$$



Using Pythagoras

$$PQ_2^2 = PQ_1^2 = 1^2 + 0.5^2$$

$$PQ_1 = \sqrt{1.25}$$

$$PQ_1 = 1.12 \text{ m}$$

$$E_p = EQ_1 + EQ_2 + EQ_q \quad \downarrow \quad \text{but } E_p = 0$$

$$EQ_1 = EQ_2 = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 57397.96 \text{ NC}^{-1}$$

$$EQ_q = \frac{kq}{r^2} = \frac{9 \times 10^9 \times q}{(1)^2} = 9 \times 10^9 q \text{ NC}^{-1}$$

$$\text{From eqn i: } 0 = 57397.96 + 57397.96 + 9 \times 10^9 q - 9 \times 10^9 q = 114795.92$$

$$\therefore q = 1.3 \times 10^{-5} \text{ C} = -1.3 \times 10^{-5} \times 10^{-6}$$

$$\therefore q = -13 \text{ C}$$

2. Electric field is a region of space in which an electric charge will experience an electric force while electric field intensity can be defined as the force per unit charge (F/q).

b. Find at Point P

$$EQ_1 = \frac{kQ_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{7^2} = 1.47$$

Section B

4a Magnetic flux: It can be defined as the strength of a magnetic field represented by lines of force.

4b $m = 9.11 \times 10^{-31} \text{ kg}$, $r = 1.4 \times 10^{-7} \text{ m}$, $B = 3.5 \times 10^{-1} \text{ weber/m}$

$q_e = -1.6 \times 10^{-19}$; but cyclotron frequency = angular speed.

$$\omega = \frac{q \cdot B}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

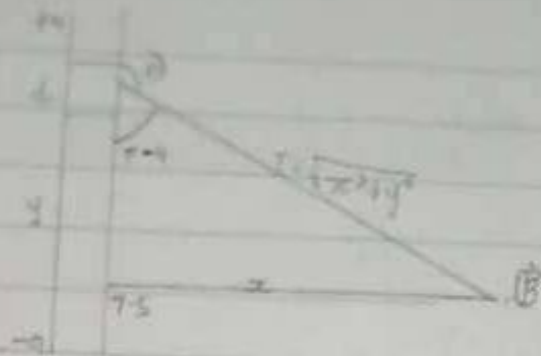
$$\omega = 6.15 \times 10^{10} \text{ rad/sec}^{-1}$$

4c The angular ^{speed or ω} is referred to as the cyclotron frequency: This is because the charge particle circulates at this angular frequency or angular speed in the type of accelerator called cyclotron. We were given the required parameters, m , B and q and we found the angular speed which is equal to the cyclotron frequency = $6.15 \times 10^{10} \text{ J}^{-1}$

5. Biot-Savart law states that the magnetic field $d\vec{B}$ due to current I flowing through a small element $d\vec{l}$ is directly proportional to the current I , the length $d\vec{l}$ and sine of the angle ϕ (which is the angle between \hat{r} and $d\vec{l}$) and inversely proportional to the square of the distance from $d\vec{l}$ to p , r^2 . Mathematically,

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \text{ where } \hat{r} = \text{unit vector}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$$



Applying the Biot-Savart law, we find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \theta}{r^2}$$

$$\sin(\pi - \theta) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{r^2}$$

from diagram, $r^2 = x^2 + y^2$.

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \theta)}{x^2 + y^2} \quad \text{--- (i)}$$

$$\text{But } \sin(\pi - \theta) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{--- (ii)}$$

Substituting (ii) into (i).

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \text{--- (iii)}$$

Using special integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x^2} = \frac{y}{x^2(x^2 + y^2)^{1/2}}$$

Equation (iii) becomes:

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2(x^2 + y^2)^{1/2}} \right]_{-a}^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2 + a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2 + a^2)^{1/2}} \right)$$

When the length $2a$ of the conductor is very great in comparison to its distance x from point P , we consider it infinitely long. That is, when a is much larger than x ,

$$(x^2 + a^2)^{1/2} \approx a, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$