

Assignment 1

$$1) \int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \sqrt{4x^2-1} + c$$

$$2) \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \frac{(\sin^{-1} x)^2}{2} + c$$

$$3) \int (\tan x)^n \sec^n x dx \Rightarrow \begin{cases} \int u = \tan x \\ \frac{du}{dx} = \sec^2 x dx \\ \frac{du}{u} = \sec^2 x dx \end{cases}$$

1) Soln

$$\text{let } u = 4x^2 - 1$$

$$u+1 = 4x^2$$

$$\frac{u+1}{4} = x^2$$

$$x = \frac{1}{2} \sqrt{u+1} \quad (\text{making } x \text{ the subject})$$

$$\frac{dx}{du} = \frac{1}{4(u+1)^{3/2}}$$

making dx the subject of the formula

$$dx = \frac{du}{4(u+1)^{3/2}}$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{2 \cdot \frac{1}{2} \sqrt{u+1}}{2} \cdot \frac{du}{4(u+1)^{3/2}} = \frac{1}{4} \cdot \frac{du}{\sqrt{u+1}}$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \int \frac{1}{4\sqrt{u}} du$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \int \frac{1}{\sqrt{u}} du$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[\frac{v^{-1/2} + 1}{1/2} \right] + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[\frac{v^{3/2}}{3/2} \right] + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{4} \left[2\sqrt{4} \right] + c$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} x \sqrt{v} + c \quad (\text{recall } v = 4x^2 - 1)$$

$$\int \frac{2x}{\sqrt{4x^2-1}} dx = \frac{1}{2} \sqrt{4x^2-1} + c$$

2) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

Soln

let v be $\sin^{-1} x$

recall the differential of $\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$dx = d \cdot (\sqrt{1-x^2}) dv$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int \frac{v}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} dv$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \int u dv$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \left[\frac{u^{1+1}}{2} \right] + c$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{1}{2} (\sin^{-1} x)^2 + c$$

3) Soln
let v be $\tan x$
 $\frac{dv}{dx} = \sec^2 x$
 $dx = \frac{dv}{\sec^2 x}$

$$\int (\tan x)^2 dx$$

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Recall d

$$\int (\tan x)^2 dx$$

$$\frac{1}{2} + 1 \Big) + e$$

$$\frac{1}{2} \Big) + e$$

$$\Big) + e$$

$$+ e \text{ (recall } v = 4x^2 - 1)$$

$$+ e$$

$$x = \frac{1}{\sqrt{1-x^2}}$$

$$-x^2 dv$$

②

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{u^2}{2} + C \text{ (recall } u = \sin^{-1} x)$$

$$\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} = \frac{(\sin^{-1} x)^2}{2} + C$$

3) Solnlet v be $\tan x$

$$\frac{dv}{dx} = \sec^2 x$$

$$dx = \frac{dv}{\sec^2 x} \text{ (making } dx \text{ the subject of the formula)}$$

$$\int (\tan x)^6 \sec^2 x dx = \int u^6 \cdot \sec^2 x \cdot \frac{dv}{\sec^2 x}$$

$$\int (\tan x)^6 \sec^2 x dx = \int u^6 dv$$

$$\int (\tan x)^6 \sec^2 x dx = \left(\frac{u^{7+1}}{7} \right) + C$$

$$\int (\tan x)^6 \sec^2 x dx = \frac{u^7}{7} + C$$

Recall $u = \tan x$

$$\int (\tan x)^6 \sec^2 x dx = \frac{(\tan x)^7}{7} + C$$