NAME: AWALA DIVINE PAUL

MATRIC NUMBER: 19/ENG05/016

DEPARTMENT: MECHATRONICS ENGINEERING

COURSE CODE: PHY102

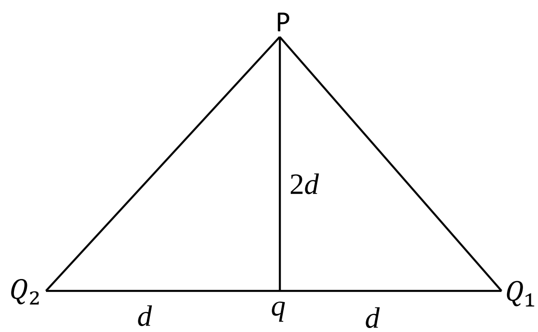
Covid-19 Holiday Assignment

QUESTION 1

(a) Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

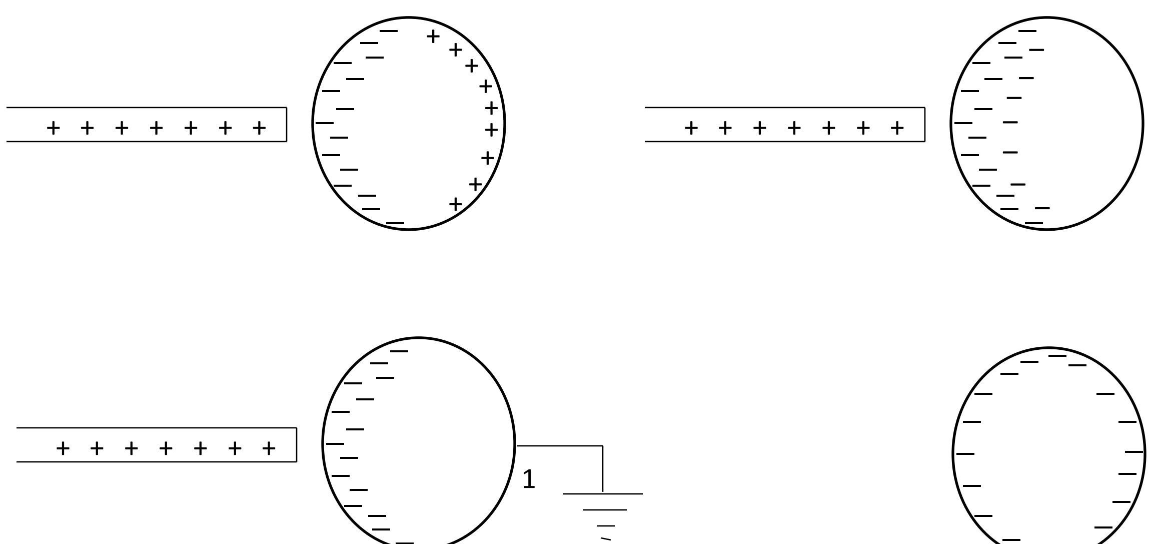
(b) Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5}C. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, calculate the charge on each sphere.

(c) Three charges were positioned as shown in the figure below. If $Q_1 = Q_2 = 8\mu C$ and d = 0.5m, determine $\boldsymbol{q}$ if the electric field at P is zero.



SOLUTION

(a) Consider a positively charged rubber rod brought near a neutral conducting sphere that is insulated so that there is no conducting path to the ground. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side farthest away from the rod. The region of sphere nearest the positively charged rod has an excess of the negative charge because of the migrations of protons away from this location. If a grounded conducting wire is then connected to the sphere, some of the protons leave the sphere and travel to the earth. If the wire to the ground is then removed, the conducting sphere is left with an excess of induced negative charge. Finally, when the rubber rod is removed from the vicinity of the sphere the induced negative charge remains on the grounded sphere and becomes uniformly distributed over the surface of the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.



(b) F = 1.0N

R = 2.0N

$F = \frac{Kq_1q_2}{r^2}$

$Fr^2 = kq_1q_2$

$\frac{Fr^2}{k} = q_1q_2$

$\frac{1.0(2)^2}{9.0 \times 10^9} = q_1q_2$

$q_1q_2 = 4.444 \times 10^{-10}$ ….. (i)

$q_1 + q_2 = 5.0 \times 10^{-5}$ ….. (ii)

$q_1\, 5.0 \times 10^{-5} - q_2$

substituting q_1 in equation (i)

$(5.0 \times 10^{-5} - q_2)q_2 = 0.444 \times 10^{-10}$

$5.0 \times 10^{-5}q_2 - (q_2)^2 = 0.444 \times 10^{-10}$

$q_2^2 - 5.0 \times 10^{-5}q_2 + 0.444 \times 10^{-10} = 0$

Using general formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $a = 1$; $b = -0.00005$; $c = 0.000000000444$

$$x = \frac{-(-0.00005) \pm \sqrt{(-0.00005)^2 - 4(1)(0.000000000444)}}{2(1)}$$

$$x = \frac{0.00005 \pm \sqrt{0.0000000025 - 0.000000001776}}{2}$$

$$x = \frac{0.00005 \pm \sqrt{0.000000000724}}{2}$$

$$x = \frac{0.00005 \pm 0.00002691}{2}$$

$$x = \frac{0.00005 + 0.00002691}{2}$$

$\boldsymbol{q_1 = 3.85 \times 10^{-5}}$**C**

$$x = \frac{0.00005 - 0.00002691}{2}$$

$\boldsymbol{q_2 = 1.15 \times 10^{-5}}$**C**

OR

$\boldsymbol{q_1 = 1.15 \times 10^{-5}}$**C**

$\boldsymbol{q_2 = 3.85 \times 10^{-5}}$**C**

(c) $E_P = E_{Q1} + E_{Q2} + E_q$

$E_{Q1} = \frac{kQ_1}{r_1{}^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = \frac{72 \times 10^3}{1.25} = 57.6 \times 10^3$ N/C

$E_{Q2} = \frac{kQ_2}{r_2{}^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{1.12^2} = \frac{72 \times 10^3}{1.25} = 57.6 \times 10^3$ N/C

$E_q = \frac{k}{r^2} = \frac{9 \times 10^9 \times q}{1^2} = 9 \times 10^9 q$

$115.2 \times 10^3 + 9 \times 10^9 q = 0$

$9 \times 10^9 q = -115.2 \times 10^3$

$q = \frac{-115.2 \times 10^3}{9 \text{ x } 109} = -12.8 \times 10^{-6} = -12.8\mu C$

QUESTION 2

(a) Distinguish between the terms: electric field and electric field intensity.

(b) A positive charge $Q_1 = 8nC$ is at the origin, and a second positive charge $Q_2 = 12nC$ is on the *x*- axis at *x* = 4m. Find

(i) the net electric field at a point P on the axis at .

(ii) the electric field at a point Q on the *y*- axis at *y* = 3m due to the charges.

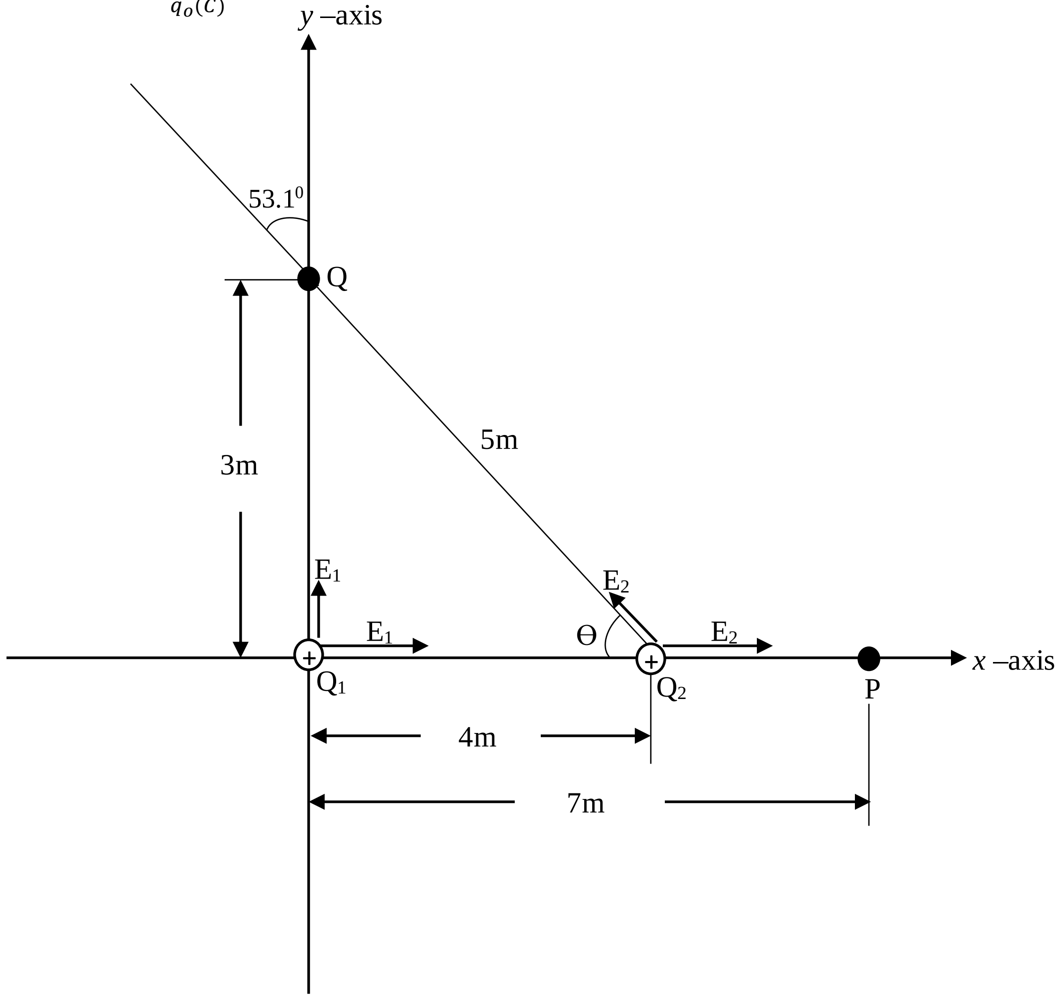
SOLUTION

(a) An electric field is a region of space in which an electric charge will experience an electric force.

Electric field strength (intensity) E, can be defined as the force per unit charge.

$E = \frac{F(N)}{q_o(C)}$

(b)



(i) $E_P = E_1 + E_2$

$E_1 = \frac{kQ_1}{{r_1}^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{7^2} = 1.47NC^{-1}$

$E_2 = \frac{kQ_2}{{r_2}^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{3^2} = 12NC^{-1}$

Field(N/C)	Angle	X – Component	Y – Component
$E_1 = 1.47$	0^0	$1.47 \times \cos0^0$	$1.47 \times \sin0^0$
		= 1.47N/C	= 0
$E_2 = 12$	0^0	$12 \times \cos0^0$	$12 \times \sin0^0$
		= 12N/C	= 0
		$\sum_X = 13.47$N/C	$\sum_Y = 0$

$$E_p = \sqrt{E_x^2 + E_y^2}$$
$$= \sqrt{13.47^2 + 0^2}$$
$$= \sqrt{181.4409 + 0}$$
$$= \sqrt{181.4409}$$
$$= 13.47NC^{-1}$$

(ii) $E_Q = E_1 + E_2$

$$E_1 = \frac{kQ_1}{{r_1}^2} = \frac{9 \times 10^9 \times 8 \times 10^{-9}}{3^2} = 8NC^{-1}$$

$$E_2 = \frac{kQ_2}{{r_2}^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{5^2} = 4.32NC^{-1}$$

Field(N/C)	Angle	X – Component	Y – Component
$E_1 = 8$	90^0	$8 \times \cos90^0$	$8 \times \sin90^0$
		= 0	= 8N/C
$E_2 = 4.32$	36.9^0	$4.32 \times \cos36.9^0$	$4.32 \times \sin36.9^0$
		= 3.45N/C	= 2.59N/C
		$\sum_X = 3.45$N/C	$\sum_Y = 10.59$N/C

$$E_Q = \sqrt{E_x^2 + E_y^2}$$
$$= \sqrt{3.45^2 + 10.59^2}$$
$$= \sqrt{11.9025 + 112.1481}$$
$$= \sqrt{124.0506}$$
$$= 11.14NC^{-1}$$

Direction

$\tan\Theta = \frac{E_y}{E_x}$

$= \frac{10.59}{3.45}$

$= 3.0696$

$\Theta = \tan^{-1}(3.0696)$

$= 71.9557$

$\Theta = 72°$

QUESTION 3

(a) State the formulation of the following identities of charges:

(i) Volume Charge density

(ii) Surface Charge density

(iii) Linear Charge density

(b) Explain with appropriate equations, the electric potential difference

(c) Two point charges Q_1 = 10μc and Q_2 = – 2μc are arranged along the x-axis at x = 0 and x = 4m respectively. Find the position along the x-axis where v = 0.

SOLUTION

(a)

(i) Volume charge density, $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

(ii) Surface charge density, $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

(iii) Linear charge density, $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

(b) $dW = F.dl$

But $F = -q_0E$

$dW = -q_oE \, . \, dl$

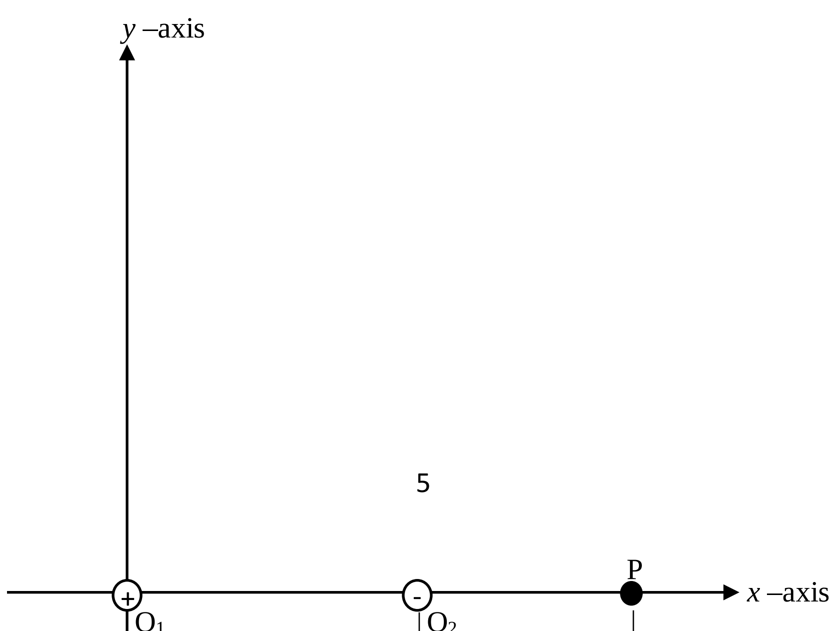
When a text charge is moved from point A to point B

$W(A \rightarrow B) = -q_0\int_A^B Edl$

$V_B - V_A = \frac{W(A \rightarrow B)Ag}{q_o}$

$\therefore V_B - V_A = -\int_A^B Edl$

(c)



Using V = $\frac{kQ1}{(x)} + \frac{KQ2}{(x\ -\ a)}$

$$V = \frac{9.0 \times 10^{9} \times 10 \times 10^{-6}}{x} - \frac{9.0 \times 10^{9} \times 2 \times 10^{-6}}{x\ -\ 4}$$

$$0 = \frac{9.0 \times 10^{9} \times 10 \times 10^{-6}}{x} - \frac{9.0 \times 10^{9} \times 2 \times 10^{-6}}{x\ -\ 4}$$

$$\frac{9.0\ \ 10^{9}\ x\ 2\ x\ 10^{-6}}{x\ -\ 4} = \frac{9.0\ x\ 10^{9}\ x\ 10\ x\ 10^{-6}}{x}$$

$2 \times 10^{-6} \times x = 10 \times 10^{-5}(x - 4)$

$0.000002x = 0.000010x - 0.00004$

$0.00004 = 0.00001x - 0.000002x$

$0.00004 = x(0.000008)$

$x = \frac{0.00004}{0.000008}$

$\mathbf{x = 5m}$

QUESTION 4

(a) What is Magnetic flux?

(b) An electron with a rest mass of $9.11\ \times\ 10^{-31}$kg moves in a circular orbit of radius $1.4\ \times\ 10^{-7}$m in a uniform magnetic field of $3.5\ \times\ 10^{-1}$ Weber/meter square, perpendicular to the speed with which electron moves. Find the cyclotron frequency of the moving electron.

(c) Discuss your answer in 4(b) above.

SOLUTION

(a) Magnetic flux is defined as the number of magnetic field lines passing through a given closed surface. It gives the measurement of the total magnetic field that passes through a given surface area.

OR

It is the strength of magnetic field represented by lines of force.

$\phi_B = B.\ A = BACos\Theta$

(b) Using $\frac{2\pi}{\omega} = \frac{2\pi m}{qB}$

$2\pi q B = 2\pi m\omega$

$$\omega = \frac{2\pi qB}{2\pi m} = \frac{qB}{m}$$

$$\omega = \frac{1.60 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$= \frac{5.6 \times 10^{-20}}{9.11 \times 10^{-31}}$$

$= 0.6147 \times 10^{11}$

$\therefore \omega = 6.15 \times 10^{10}\ ms^{-1}$

(c) The angular speed ω, which is = $6.15 \times\ 10^{10}\ ms^{-1}$ is often referred to as the cyclotron frequency because the charge particle (electron) circulates at this angular frequency or angular speed in the type of accelerator called cyclotron.

QUESTION 5

(a) State the Biot – Savart Law.

(b) Using the Biot-Savart Law, show that the magnitude of the magnetic field of a straight current-carrying conductor is given as:

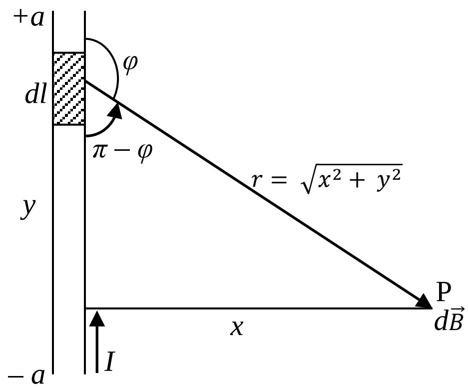
$$B = \frac{\mu_0 I}{2\pi r}$$

SOLUTION

(a) Biot – Savart law states that the magnetic field produced by a current-carrying conductor is directly proportional to the current, I and the length element dL and inversely proportional to the square of the distance of separation.

$$B = \frac{\mu_0 I}{2\pi x}$$

(b) Magnetic field of a straight current carrying conductor.



Applying the Biot – Savart Law, we can find the magnitude of the field $d\vec{B}$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{dl\ Sin\emptyset}{r^2}$$

$\text{Sin}(\pi - \varphi) = Sin\varphi$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{dl\ Sin(\pi - \varphi)}{r^2}$$

From diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a}\frac{dl\ Sin(\pi - \varphi)}{x^2 + y^2} \qquad \text{…… (i)}$$

But $\text{Sin}(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{\frac{1}{2}}}$ …… (ii)

Substituting (ii) into (i), we have

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{\frac{1}{2}}}$$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)^{\frac{3}{2}}}$$

Recall $dl = dy$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{\frac{3}{2}}}$$

$$B = \frac{\mu_o I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{\frac{3}{2}}} dy$$

$$B = \frac{\mu_o I x}{4\pi}\int_{-a}^{a} \frac{1}{(x^2+y^2)^{\frac{3}{2}}} dy \qquad \text{....... (iii)}$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{\frac{3}{2}}} = \frac{1}{x^2}\frac{y}{(x^2+y^2)^{\frac{1}{2}}}$$

Equation (iii) therefore becomes

$$B = \frac{\mu_o I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{\frac{1}{2}}}\right]_{-a}^{a}$$

$$B = \frac{\mu_o I x}{4\pi}\left(\frac{2a}{(x^2+y^2)^{\frac{1}{2}}}\right)$$

$$B = \frac{\mu_o I}{4\pi x}\left(\frac{2a}{(x^2+y^2)^{\frac{1}{2}}}\right)$$

When the length 2a of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long.

That is, when a is much larger than x, $(x^2 + y^2)^{\frac{1}{2}} \cong a,$ as $a \to \infty$

$$\therefore B = \frac{\mu_o I}{2\pi x}$$

QUESTION 6

(a) Explain the practical application of Faraday's Law in the production of sound in an electric guitar.

(b) A coil consists of 300 turns of wire having a total resistance of 2.0Ω. Each turn is a square of side 10cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 10T in 0.5 sec,

(i) What is the magnitude of the induced e.m.f in the coil while the field is changing?

(ii) What is the magnitude of the induced current in the coil while the field is changing?

(c) The plane of a rectangular coil of dimensions 5cm by 8cm is perpendicular to the direction of a magnetic field B. If the coil has 75 turns and a total resistance of 8Ω, at what rate must the magnitude of the B change in order to induce a current of 0.1A in the windings of the coil?

SOLUTION

(a) The coil in this case called the pickup coil is placed near the vibrating guitar string which is made of a metal that can be magnetized. A permanent magnet inside the coil magnetizes the profurn of the string nearest the coil. When the string vibrates at some frequency, its magnetized segment produces a charging magnetic flux trough the coil. The changing flux induces as e.m.f in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, which produces the sound waves we hear.

(b) *(i)* $|\xi| = N\frac{d\phi_B}{dt}$

$= \text{N}\frac{\Delta\phi_B}{\Delta t}$

$= \text{N}\frac{(\phi_{Bt2} - \phi_{Bt1})}{t_2 - t_1}$

$= 300\,\frac{(10 - 0)}{0.5}$

= 6000V

(ii) $\text{I} = \frac{\xi}{R}$

$= \frac{6000}{2}$

= 3000A

(c) Using $\xi = \text{-N}\frac{d\phi_B}{dt}$

But ξ = IR

I = 0.1A

R = 8Ω

$\xi = 0.1 \times 8$

= 0.8V

$\frac{d\phi_B}{dt} = \frac{\xi}{N} = \frac{0.8}{75}$

$\frac{d\phi_B}{dt} = 0.010667$

$\therefore\ \frac{d\phi_B}{dt} = 1.07 \times 10^{-2}\ \text{TS}^{-1}$