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COMPUTER ENGINEERING (19/ENG02/054)
PHY 102 (COVID-19 HOLIDAY ASSIGNMENT)
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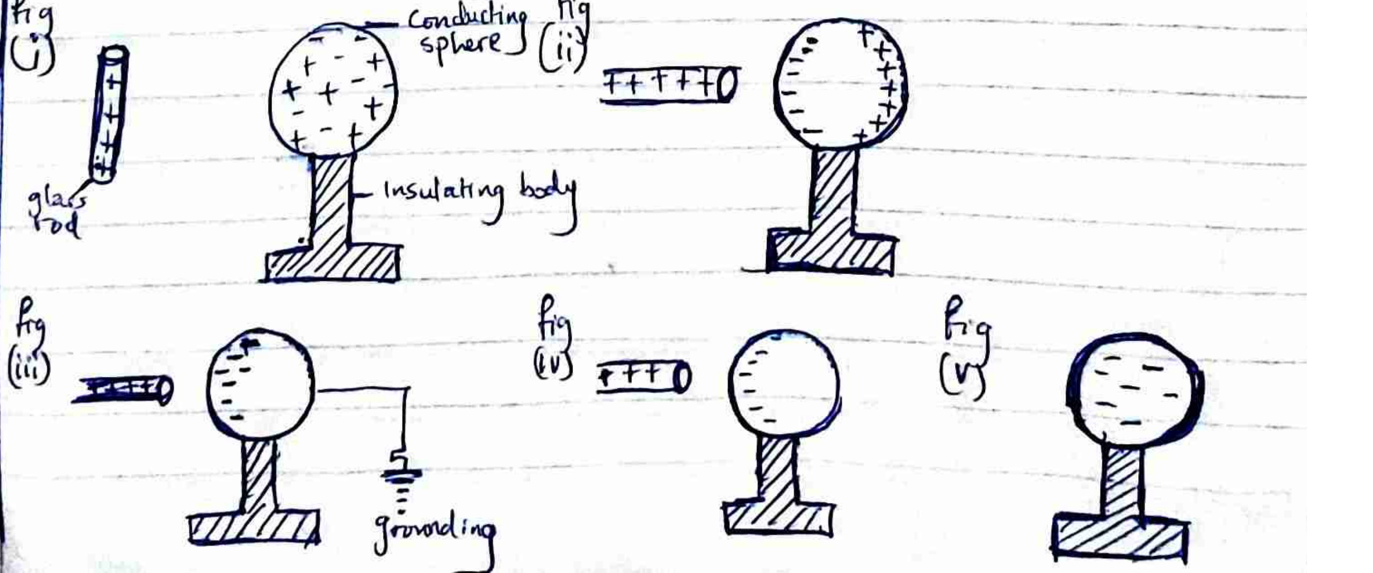
1a CHARGING BY INDUCTION

Induction is the process by which an object can attain charges without being in contact with the host charge. In this experiment, I will be showing you how to ~~charge~~ charge a sphere/ induce negative charges on a sphere.
First you will need a conducting sphere that is neutral/uncharged. The sphere must be insulated such that there is no conducting path with the ground. Next, you will need a positively charged rod, such as a glass rod. The apparatus will be set as in figure (I) below. Next, you will place the positive rod close to the neutral sphere. This causes a redistribution of charges in the sphere such that the negative charges are closer to the rod, while the positive charges repel and move further away from the glass rod. Due to the presence of the rod, excess positive charge that is far apart can be grounded (Earthened). Grounding is the process of ~~trans~~ transferring excess charge on an object to the earth. When the positive charge is grounded and the glass rod is moved far away from the conducting sphere, the negative charges that has been induced, becomes uniformly distributed over the surface of the sphere.



1b $q_1 + q_2 = 5 \times 10^{-5}\,C$

$F = 1N$

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

$r = 2m$

The charge on the sphere $\Rightarrow$ $q_1 q_2 = \frac{Fr^2}{K}$

$$q_1 q_2 = \frac{1 \times 2 \times 2}{9 \times 10^9} = 4.44 \times 10^{-10}\,C$$

We can the conclude that $q_1 + q_2 = 5 \times 10^{-5}\,C$ — eqn I

and $q_1 \cdot q_2 = 4.44 \times 10^{-10}\,C$ — eqn II

from eqn I, we can say that $q_1 = 5\times10^{-5}C - q_2$ — eqn III

put eqn III into eqn II $\Rightarrow$ $(5\times10^{-5}C - q_2)q_2 = 4.4\times10^{-10}C$

$5\times10^{-5}q_2 - q_2^2 = 4.4\times10^{-10}C$

$q_2^2 - (5\times10^{-5}q_2) + 4.4\times10^{-10} = 0$ $\Rightarrow$ Quadratic equation

After using the formula $q_2 = \frac{-b \pm (b^2 - 4ac)}{2a}$

where $a = 1$, $b = -5\times10^{-5}$, $c = 4.4\times10^{-10}$

we have that $q_2 = 1.155\times10^{-5}C$ or $3.845\times10^{-5}C$

recall that $q_1 + q_2 = 5\times10^{-5}C$

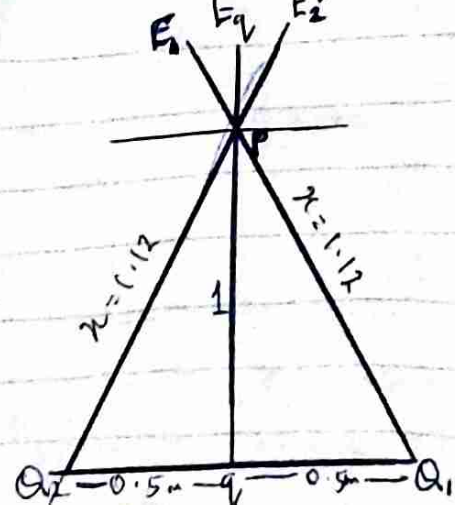
$\therefore$ $q_1 = 5\times10^{-5} - 1.155\times10^{-5} = 3.845\times10^{-5}C$

$q_2 = 5\times10^{-5} - 3.845\times10^{-5} = 1.155\times10^{-5}C$

$\therefore$ $q_1 = 3.845\times10^{-5}C$ and $q_2 = 1.155\times10^{-5}C$ (or vice versa)

1c $Q_1 = Q_2 = 8\mu C$ (determine q is electric field at $P = 0$)

$d = 0.5m$



$r^2 = 1^2 + 0.5^2$

$r = \sqrt{1.25}$

$r = 1.12$

$\tan\theta = \frac{1}{0.5}$

$\theta = \tan^{-1}(2)$

$\theta = 63.4°$

$$E_1 = \frac{kq_1}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2} = 57397.9592$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2} = 57397.9592$$

$$E_q = \frac{kq}{r^2} = \frac{9\times10^9 \times q}{1^2} = 9\times10^9 q$$

Vector & Magnitude	Angle	x - Component	y - Component
$E_1 = 57397.9592$	$63.4°$	$E_1 \cos\theta = 25700.4579$	$E_1 \sin\theta = 51322.6284$
$E_2 = 57397.9592$	$63.4°$	$E_2 \cos\theta = 25700.4579$	$E_2 \sin\theta = 51322.6284$
$E_q = 9\times10^9 q$	$90°$	$E_q \cos\theta = 0$	$E_q \sin\theta = 9\times10^9 q$
		$\Sigma x = 0$	$\Sigma y = 102645.2568 + 9\times10^9 q$

$$\Sigma q = \text{Magnitude} = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$\Sigma q = \sqrt{0^2 + (102645.2568)^2}$$

$$\Sigma q = 102645.2568 + 9\times10^9 q$$

at $\Sigma q = 0$

$$0 = 102645.2568 + 9\times10^9 q$$

$$\frac{-102645.2568}{9\times10^9} = \frac{\cancel{9\times10^9} q}{\cancel{9\times10^9}}$$

$$q = 1.14\times10^{-5} C$$

$$q = \underline{\underline{11.4 \mu C}}$$

3ai Volume charge density, $\rho = \frac{dQ}{dv} \rightarrow dQ = \rho dv$

ii Surface charge density, $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

iii Linear charge density, $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

3b) ELECTRIC POTENTIAL DIFFERENCE

The Electric Potential difference between two points in an electric field can be defined as the workdone per unit charge against electric forces when a charge is transported from one point to another. It is measured in Volts (v) or Joules per columb (J/c) and It is a Scalar quantity.

Elemental workdone dW is given as:

$$dW = F \cdot dL \quad \text{— (I)}$$

But $F = q_0 E$ — (II)

Substituting eqn II into eqn I we have that

$$dW = q_0 E dL \quad \text{— (III)}$$

Total Workdone in moving the test charge from A to B is:

$$W(A \cap B)_{Ag} = -q_0 \int_A^B E dL \quad \text{— (IV)}$$

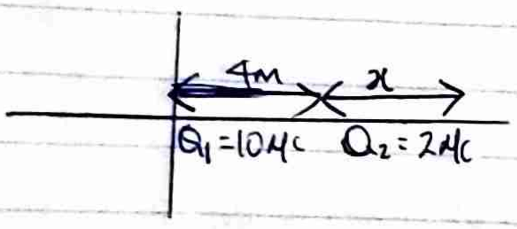
from the definition of electric potental difference, it follows that

$$V_B - V_A = \frac{W(A \cap B)_{Ag}}{q_0} \quad \text{— (V)}$$

Putting eqn IV into eqn (V) yields:

$$V_B - V_A = -\int_A^B E dL \quad \text{— (VI)}$$

3c)



$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right]$$

let $V_P = 0$

$$\therefore 0 = 9 \times 10^9 \left[\frac{10 \times 10^{-6}}{4+x} + \frac{(-2 \times 10^{-6})}{x}\right]$$

$$0 = \frac{90000}{4+x} - \frac{18000}{x}$$

Multiply through by $(4+x)(x)$

$$(4+x)(x)(0) = \frac{90000\cancel{(4+x)}(x)}{\cancel{4+x}} - \frac{18000(4+x)\cancel{(x)}}{\cancel{x}}$$

$$90000x - 18000(4+x) = 0$$

$$90000x - 72000 - 18000x = 0$$

$$90000x - 18000x = 72000$$
$$72000x = 72000$$
$$x = 1$$
$$4 + x = 4 + 1 = 5m$$
$\therefore$ The position on the x-axis where $V = 0$ is $\underline{5m}$

* SECTION B

4a Magnetic flux is defined as the strength of the magnetic field which can be represented by lines of forces. It is represented by the symbol "Φ" and mathematically given as $\Phi = B \cdot dA$

4b
$M = 9.11 \times 10^{-31}$ kg
$r = 1.4 \times 10^{-7}$ m
$B = 3.5 \times 10^{-1}$ weber/m^2
Cyclotron frequency = angular speed

$$W = \frac{v}{r} = \frac{qB}{M}$$

$$W = \frac{qB}{M} = \frac{-1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$$

$$W = \underline{-6.15 \times 10^{10} \text{ rad/sec}}$$

4c Mass of electron $= 9.11 \times 10^{-31}$, Radius $= 1.4 \times 10^{-7}$ m are the given parameters in the above question. (including magnetic field $= 3.5 \times 10^{-1}$ weber/m^2)
We were asked to find cyclotron frequency, which is equals to angular speed.
Recall that angular speed $W = \frac{v}{r} = \frac{qB}{M}$

Where M = mass of electron, B = magnetic field, and q is -1.6×10^{-19} (constant)
we substitute the values to their corresponding letters
Therefore $\frac{-1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = \underline{-6.15 \times 10^{10} \text{ rad/sec}}$

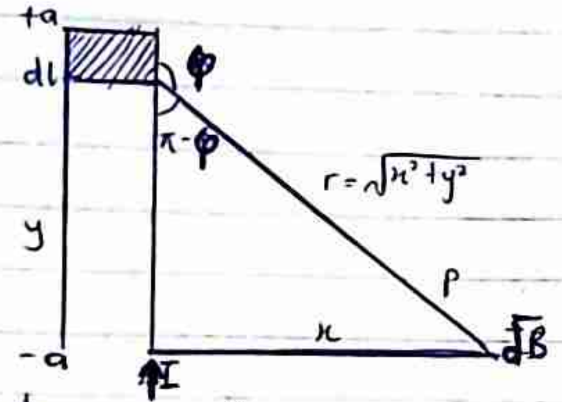
N.B: Having a unit of rad per second is equal to the unit of frequency dimensionally and it means that the charged particle electron circulates in a negative or opposite direction at the angular frequency.

5a Biot-savart law states that the magnetic field is directly proportional to the product permeability of free space (μ), the current (I), the change in length, the radius and inversely proportional to the square of the radius (r^2). It can be represented mathematically by

$$dB = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$$

where μ_0 is a constant called permeability of free space $= 4\pi \times 10^{-7}$ T.m/A and the unit of B is weber / meter square.

5b



A Section of a straight current carrying Conductor

Applying the Biot-Savart law, we find the magnitude of the field $\vec{dB}$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin\theta}{r^2}$$

$$\sin(\pi - \varphi) = \sin\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$$

from the diagram, $r^2 = x^2 + y^2$ (pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2} \quad \text{— (I)}$$

$$\text{But } \sin(\pi - \varphi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \text{— (II)}$$

Substituting eqn II into I, we have

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$ $\therefore B = \frac{\mu_0 I}{4\pi}\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}}\,dy$

$$B = \frac{\mu_0 I x}{4\pi}\int_{-b}^{a} \frac{1}{(x^2+y^2)^{3/2}}\,dy \quad \text{--- (IV)}$$

Using special Integrals: $\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2}\cdot\frac{y}{(x^2+y^2)^{1/2}}$

Equation (IV) therfore becomes

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+a^2)^{1/2}}\right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi}\left(\frac{2a}{x^2(x^2+a^2)^{1/2}}\right)$$

$$B = \frac{\mu_0 I}{4\pi x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length 2a of the conductor is very great in comparison to its distance x from point P, we consider it infinitely long. That is, when a is much larger than x,

$$(x^2+a^2)^{1/2} \cong a, \text{ as } a \longrightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y-axis. Thus, at all points in a circle of radius r, around the conductor, the magnitude of B is $\Rightarrow$ $B = \frac{\mu_0 I}{2\pi r}$ --- $*$

Equation ($*$) defines the magnitude of the magnetic field of flux density B near a long straight current carrying conductor