19/ENG02/058
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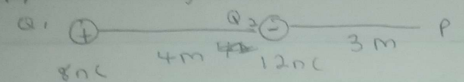
2) An electric field is a region of space under which an electric charge experiences an electric force, while electric field intensity is the electric force per unit charge.

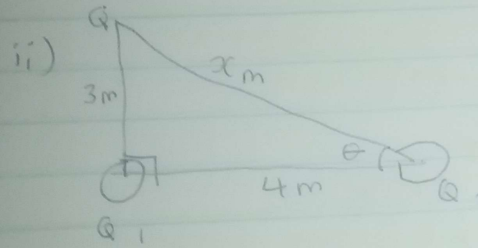
b) i) $Q_1 = 8nC$, $Q_2 = 12nC$



$$E_N = \frac{KQ_1}{r_1^2} + \frac{KQ_2}{r_2^2} = \frac{9\times10^9 \times 8\times10^{-9}}{7^2} + \frac{9\times10^9\times12\times10^{-9}}{3^2}$$

$$E_N = 1.47 + 12 = 13.47 N/C$$

ii)



$x = \sqrt{3^2 + 4^2} = 5m$

$\theta = \tan^{-1}(3/4) = 36.87°$

$$E_1 = \frac{9\times10^9 \times 8\times10^{-9}}{3^2} = 8 N/C \qquad E_2 = \frac{9\times10^9 \times 12\times10^{-9}}{5^2} = 4.32 N/C$$

Vector	x com	y com
E_1	8 cos 90	8 sin 90
E_2	− 4.32 cos 36.87	4.32 sin 36.87
Σ =	−3.456	10.592

$$E_N = \sqrt{(-3.456)^2 + (10.592)^2} = \sqrt{124.134}$$
$$= 11.14 N/C$$

3a) i) Volume charge denisty $\rho = dQ/dv$

ii) Surface charge denisty $\sigma = dQ/dA$

iii) Linear charge density $\lambda = dQ/dL$

b) Electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It measure in volt (V) or Joules per coulomb (J/C)

$dW = F.dL$

$F = -q.E$

$dW = -q_0 E dL$

$W(A - B) = -q_0 \int_A^B E dL$

from the definition

$V_B - V_A = W(A \rightarrow B) Ag / q_0$

$\therefore V_{B-A} = -\int_A^B E dL$

4a) Magnetic flux is defined as the strength of magnetic field represented by lines of force. It is usually represented by the ϕ

5) The Biot - Savart is law based on the following Observations for magnetic field $d\vec{B}$ at a point p associated with length element $d\hat{l}$ of a wire carrying a steady current.

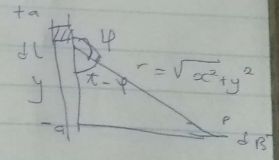
b) $\vec{dB} = \frac{\mu_o I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$ — ①

$\mu_o = 4\pi \times 10^{-7}$ T.m/A

$\vec{B} = \frac{\mu_o}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$ — ②

$B = \frac{\mu_o}{4\pi} \int \frac{I dl \sin\theta}{r^2}$ — 3

$B = \frac{\mu_o I}{4\pi} \int_{-a}^{a} \frac{dl \sin\varphi}{r^2}$



$\sin(\pi - \varphi) = \sin\theta$

$\therefore B = \frac{\mu_o I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{r^2}$

$r^2 = x^2 + y^2$ (pythagoras theorem)

$B = \frac{\mu_o I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \varphi)}{x^2 + y^2}$ ... (*)

But $\sin(\pi - \varphi) = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{(x^2+y^2)^{1/2}}$ ... (**)

substituting (**) into (*), we have

$B = \frac{\mu_o I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)(x^2+y^2)^{1/2}}$

$B = \frac{\mu_o I}{4\pi} \int_{-a}^{a} dl \frac{x}{(x^2+y^2)^{3/2}}$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy \quad \cdots (xxx)$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy$$

Using special integrals:

$$\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x^2} \frac{y}{(x^2+y^2)^{1/2}}$$

(xxx) ∴ B becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{y}{x^2 (x^2+y^2)^{1/2}} \right]_{-a}^{a}$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2 (x^2+a^2)^{1/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{1/2}} \right)$$

$$(x^2+a^2)^{1/2} \cong a, \text{ as } a \to \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$