7TH APRIL, 2020.

AKPOFURE TESE

19/MHS01/077

100 LEVEL

MEDICINE AND SURGERY

MEDICINE AND HEALTH SCIENCES

PHY 102: ELECTRICITY, MAGNETISM AND MODERN PHYSICS

ASSIGNMENT

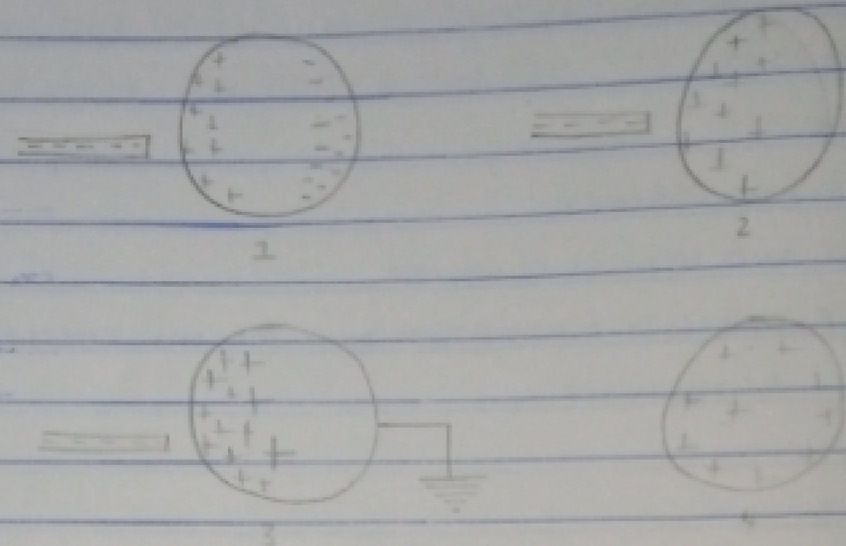
Question 1

a Explain with the aid of a diagram how you can produce a negatively charged sphere by method of induction.

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction.

Consider a negatively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the electrons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some electrons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to the negatively charged rod has an excess of positive charge because of the migration of electrons away from this location. If a grounded conducting wire is then connected to the sphere some of the electrons leave the sphere and travel to Earth. If the wire to ground is then removed, the conducting sphere is left with an excess of induced positive charge.

Finally, when the rubber rod is removed from the vicinity of the sphere, the induced positive charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.



Production of charges by induction

Question 1 (b)

b Each of two small spheres is charged positively, the combined charge being $5.0 \times 10^{-5}C$. If each sphere is repelled from the other by a force of 1.0N when the spheres are 2.0m apart, calculate the charge on each sphere.

Solution

Force, $F = 1.0N$

Radius, $r = 2.0m$

$q_1 + q_2 = 5.0 \times 10^{-5}C$

$K = 9 \times 10^9 \, Nm^2/C$

$q_1 = 5.0 \times 10^{-5}C - q_2$

$$1.0 = \frac{(9 \times 10^9)(5 \times 10^{-5} - q_2)(q_2)}{2^2}$$

$(9 \times 10^9)(5.0 \times 10^{-5} - q_2)(q_2) = 4$

$(4.5 \times 10^5 - 9 \times 10^9 q_2)(q_2) = 4$

$4.5 \times 10^5 q_2 - 9 \times 10^9 q_2^2 = 4$

$\Rightarrow 9 \times 10^9 q^2 - 4.5 \times 10^5 q_2 + 4 = 0$

Quadratically

$a = 9 \times 10^9$, $b = -4.5 \times 10^5$, $c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4.5\times10^5) \pm \sqrt{(-4.5\times10^5)^2 - 4(9\times10^9)(4)}}{2(9\times10^9)}$$

$$x = \frac{4.5\times10^5 \pm \sqrt{(2.025\times10^{11}) - 1.44\times10^{11}}}{1.8\times10^{10}}$$

$$x = \frac{4.5\times10^5 \pm \sqrt{5.85\times10^{10}}}{1.8\times10^{10}}$$

$$x = \frac{4.5\times10^5 \pm 2.42\times10^5}{1.8\times10^{10}}$$

$$x = \frac{4.5\times10^5 + 2.42\times10^5}{1.8\times10^{10}} \quad \text{or} \quad x = \frac{4.5\times10^5 - 2.42\times10^5}{1.8\times10^{10}}$$

$$x = 3.84\times10^{-5} \quad \text{or} \quad x = 1.16\times10^{-5}$$

$$\therefore q_1 = 3.84\times10^{-5}\,C \qquad q_2 = 1.16\times10^{-5}\,C$$

Question 1 (c)

Solution

$$\tan\theta = \frac{Opp}{Adj}$$

$$\theta = \tan^{-1}(1/0.5)$$

$$\theta = 63.43^\circ$$

$$E_2 = \frac{Kq_2}{r^2} = \frac{9\times10^9 \times 8\times10^{-6}}{(1.12)^2} = 57397.95918$$

$$E_1 = E_2 = 57397.95918$$

$$E_q = \frac{Kq}{r^2} = 9\times10^9 \times q = 9\times10^9 q$$

Vector	Angle	x-component	Y-component
$E_1 = 57397.95918$	63.4	25700.45785	51322.62839
$E_2 = 57397.95918$	63.4	-25700.45785	51322.62839
		$E_x = 0$	$E_y = 102645.2568$

$Eq = \sqrt{\Sigma Ex^2 + \Sigma Ey^2}$

$Eq = \sqrt{0^2 + (102645.2568)^2}$

$Eq = 0 + 102645.2568$

$q = \frac{Eq}{9 \times 10^7} = \frac{102645.2568}{9 \times 10^7}$

$q = 1.14 \times 10^{-5} C$

$q = 11.4 \times 10^{-6} C$

$q = 11.4 \mu C$

Question 4.

a) Magnetic flux is defined as the strength of magnetic field repres by lines of force. It is represented by the symbol 'Ø'.

b. $m = 9.11 \times 10^{-31} kg$

$r = 1.4 \times 10^{-7} m$

$B = 3.5 \times 10^{-1}$ weber$/m^2$

$q = 1.6 \times 10^{-19} C$

The cyclotron frequency is also called angular speed (ω)

$\omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = \frac{5.6 \times 10^{20}}{9.11 \times 10^{-31}}$

$\omega = 6.15 \times 10^{10} rads^{-1}$.

∴ cyclotron frequency $= 6.15 \times 10^{10} rad s^{-1}$

c. In the equation, we were given

mass of electron $(m) = 9.11 \times 10^{-31} kg$

a radius of $1.4 \times 10^{-1} m$

magnetic field of 3.5×10^{-1} weber$/meter^2$

The cyclotron frequency is often referred to as angular speed (ω) because it is a frequency of an accelerator called cyclotron.

Using the formula $\omega = v/r = qB/m$, we can derive the cyclotron frequency.

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{y}{x^2(x^2+y^2)^{1/2}}\right]$$

$$B = \frac{\mu_0 I x}{4\pi}\left[\frac{2a}{x^2(x^2+a^2)^{1/2}}\right]$$

$$B = \frac{\mu_0 I}{4\pi \cdot x}\left(\frac{2a}{(x^2+a^2)^{1/2}}\right)$$

When the length, $2a$ of the conductor is very great in comparison to its distance x from point P, we consider it indefinitely long. That is 'a' is much larger than x.

$$(x^2+a^2)^{1/2} \sim a \text{ as } a \longrightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y-axis, thus at all points in a circle of radius r, around the conductor, the magnitude of B is:

$$B = \frac{\mu_0 I}{2\pi r}$$

This equation defines the magnetic field of flux density B near a long, straight, current-carrying conductor.

Question 5

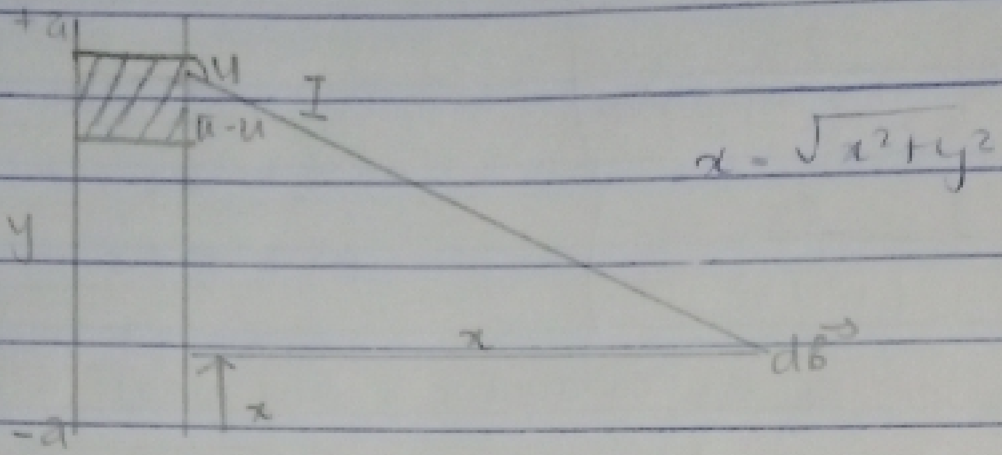
(a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ), the current (I), the change in length (dl) and the radius, and inversely proportional to the square of the radius (r^2). Mathematically,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

where μ_0 is a constant called permeability of free space

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m / A$$

b



A section of a straight current carrying conductor

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin \varphi}{r^2}$$

$$\sin(\pi - \mu) = \sin \theta$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \mu)}{r^2}$$

From the diagram above, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl \sin(\pi - \mu)}{x^2 + y^2}$$

But $\sin(\pi - \mu) = \dfrac{x}{\sqrt{x^2+y^2}} = \dfrac{x}{(x^2+y^2)^{1/2}}$

By substitution, we have:

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl\ x}{(x^2+y^2)(x^2+y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{dl\ x}{(x^2+y^2)^{3/2}}$$

Recall that $dl = dy$; $\therefore$ $B = \dfrac{\mu_0 I}{4\pi} \displaystyle\int_{-a}^{a} \frac{x}{(x^2+y^2)^{3/2}} dy$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^{a} \frac{1}{(x^2+y^2)^{3/2}} dy$$

Using special integrals: $\displaystyle\int \frac{dy}{(x^2+y^2)^{3/2}} = \frac{1}{x} \cdot \frac{y}{(x^2+y^2)^{3/2}}$

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Question 2

(a) Distinguish between the terms electric field and electric field intensity.

Electric field is a region where an electric force is experienced by a charged body while an electric field intensity at any point is defined as the force per unit positive charge (q) at that point ($E = F/q$). Electric field is represented by electric lines of force while electric field intensity is due to several charges.

(b) A positive charge $Q_1 = 8nC$ is at the origin and a second positive charge $Q_2 = 12nC$ is on the x-axis at $x = 4m$. Find:

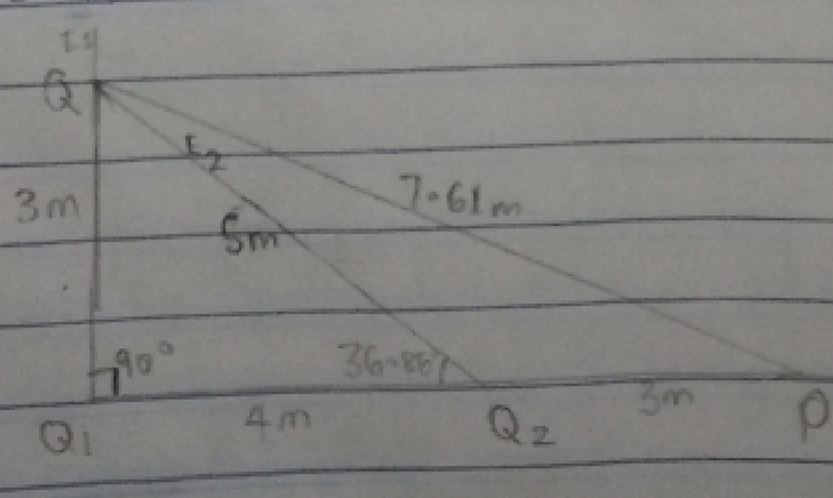
(i) the net electric field at a point P on the x axis at $x = 7m$.

(ii) the electric field at a point Q on the y axis at $y = 3m$ due to the charges.

Solution

$Q_1 = 8nC = 8 \times 10^{-9}C$

$Q_2 = 12nC = 12 \times 10^{-9}C$



(i) $\tan\theta = \text{Opp}/\text{Adj} = 3/4$

$$\theta = \tan^{-1}(3/4)$$

$$\theta = 36.9^\circ$$

$$E_{net} = E_1 + E_2$$

$$E_1 = \frac{kq}{r^2} = \frac{9\times10^9 \times 3\times10^{-9}}{7^2} = 1.469\,N/C$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9\times10^9 \times 12\times10^{-9}}{3^2} = 12\,N/C$$

$$E_{net} = 12 + 1.469 = 13.469\,N/C$$

$$E_{net} \simeq 13.5\,N/C$$

(ii) $E_2 = \frac{kq_1}{r^2} = \frac{9\times10^9 \times 12\times10^{-9}}{5} = 4.32\,N/C$

Vector	Angle	x Component	Y component
$E_1 = 8\,N/C$	90°	0	8
$E_2 = 4.32\,N/C$	36.9°	-3.45	2.59
		$\Sigma E_x = -3.45$	$\Sigma E_y = 10.59$

$$E_{net} = \sqrt{\Sigma E_x^2 + \Sigma E_y^2}$$

$$= \sqrt{(-3.45)^2 + (10.59)^2}$$

$$= 11.14\,N/C$$